Income Taxation over the Business Cycle with Wage Rigidities*

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Abstract

We study how optimal income taxes behave over the business cycle in the presence of downward rigid wages. This friction implies the existence of a price floor in the labor market. Additionally, it introduces a pecuniary externality since agents fail to recognize that their current decisions affect the price floor in the next period. We consider a standard neoclassical general equilibrium model in which the government can only tax labor and capital income. A Ramsey planner chooses the sequence of labor and capital income taxes to finance an exogenous sequence of government spending while recognizing that the current wage affects the lower bound on wages in the next period. We derive analytical results regarding the optimal labor and capital income tax rates. In a version of our model without downward rigid wages, the optimal labor income tax is constant over the business cycle and the optimal capital income tax is exactly equal to zero. We find that in the presence of downward rigid wages, the optimal labor income tax is not constant over the business cycle. It is, on average, higher when the wage is at the lower bound, because it is possible to raise tax revenue without introducing additional distortions. When the wage is above the lower bound, the labor income tax is, on average, lower. We also find that the capital income tax can be positive or negative. Finally, we solve a numerical example to illustrate these properties and conclude that the optimal labor income tax behaves counter-cyclically to output and that the optimal capital income tax behaves pro-cyclically to output.

Keywords: Optimal fiscal policy, downward wage rigidity, counter-cyclical labor income tax

JEL Codes: E24, E32, H21, J64

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1 Introduction

The classical result in the literature of optimal taxation is the Chamley-Judd result after Judd (1985) and Chamley (1986). This result states that capital income should not be taxed in the long-run. Over the business cycle, optimal fiscal policy is characterized by a similar behavior. In a stochastic neoclassical growth model calibrated to the U.S. economy, Chari, Christiano, and Kehoe (1991) show that optimal fiscal policy over the business cycle exhibits two properties: (i) the optimal labor income tax is roughly constant; and (ii) the optimal ex-ante capital income tax is approximately equal to zero.[1] There is, however, a class of utility functions such that uniform labor income taxation and zero capital income tax rates are optimal. We say that this class of utility functions represents preferences that are standard preferences in macroeconomics. They are characterized by additive separability in consumption and leisure, a constant inter-temporal elasticity of substitution of consumption, and a constant Frisch elasticity of labor supply (see Zhu (1992), Proposition 4).

In this paper, we augment the standard framework where these results hold and study how labor market frictions affect optimal fiscal policy. Particularly, we are interested in downward rigid wages as in Schmitt-Grohé and Uribe (2016) and Wolf (2020). This friction is characterized by the fact that the wage in every period cannot fall below an endogenously determined lower bound, which corresponds to a fraction of the wage in the previous period. Since agents are atomistic, they fail to recognize that their current decisions affect the lower bound on wages in the next period. Therefore, there is a pecuniary externality in our environment. The costs of this friction can be high, since the existence of a price floor in the labor market makes it possible to have involuntary unemployment along the equilibrium path for a given arbitrary fiscal policy.[2]

To address these problems, optimal fiscal policy cannot be used as under flexible prices. Instead, it must change along the business cycle depending on whether the equilibrium wage is at the

[1] For more references on these results, see, for example, Chari, Christiano, and Kehoe (1994), Chari and Kehoe (1999), and Chari, Nicolini, and Teles (2020).

[2] Our environment is a representative agent environment. Therefore, there is no involuntary unemployment in the strict sense that some agents wish to work at the given wage rate but cannot. Instead, our environment is one in which there might be underemployment along the equilibrium path.

lower bound or not, even if preferences are standard preferences in macroeconomics. We show that when the current wage rate is above the lower bound, the optimal labor income tax is used to lower the current wage, which reduces the exposure to the endogenous lower bound in the future. A lower than average labor income tax achieves this goal, because it expands labor supply, resulting in a lower equilibrium wage. However, if the current wage rate is at the lower bound, the optimal labor income should be set in a way that the pre-tax wage – the lower bound – clears the labor market. This requires choosing a higher than average labor income tax rate. The interpretation is that a higher labor income tax rate in this situation does not change the labor market outcome, making it possible to raise revenue without introducing additional distortions. Therefore, we show that the labor income tax should be lower when the current wage is above its lower bound and it should be higher when the current wage is at the lower bound. The capital income tax also plays a role in mitigating the effects of downward rigid wages because the marginal product of labor depends positively on the stock of capital. If the current wage is at the lower bound, the (*ex-ante*) capital income tax rate collected in the current period is lower. Since this event was assigned a positive probability in the previous period, the *ex-ante* capital income tax should be used to affect investment and, as a result, the stock of capital. A lower capital income tax rate induces households to invest more in capital, leading to a higher marginal product of capital in the current period, thus offsetting the negative effects of the current lower bound on wages. But if in the current period there is a positive probability that the wage is going to be at the lower bound in the next period, the *ex-ante* capital income tax rate collected in the current period should be higher. The interpretation is that setting a higher *ex-ante* capital income tax rate in the previous period, reduces the return on investment in physical capital. This, in turn, contributes to a lower marginal product of labor in the current period since lower investment results in a lower capital stock, thus reducing the exposure to the endogenous lower bound on wages in the future. We perform a simple calibration exercise to the U.S. economy and conclude that these features translate into an optimal labor income tax rate that is counter-cyclical to output and an optimal capital income tax rate that is pro-cyclical to output.

We derive these results in a standard framework in the literature of optimal fiscal policy. We consider a real dynamic stochastic general equilibrium model with endogenous labor, capital accumulation, complete markets, and a government that only has access to a state-contingent labor income taxes and an *ex-ante* capital income taxes. However, we introduce a friction in the labor market in the form of downward rigid wages. We borrow the specification from Schmitt-Grohé and Uribe (2016), though we impose that real wages are downward rigid.[3] In this environment, downward rigid wages effectively work as an endogenous price floor in the labor market, which is occasionally binding. When the wage is at the lower bound, the quantity of labor that solves the firms' profit-maximization problem is less than the quantity of labor that solves the households' *unconstrained* utility-maximization problem[4] and there is underemployment. Moreover, the quantity of labor worked is determined by the firms' demand. We study the general equilibrium effects of downward rigid wages by introducing an additional constraint on the households' consumption set. This constraint states that households cannot choose a quantity of labor above some exogenously given level, which, in equilibrium, represents the quantity of labor that solves the firms' profit-maximization problem. This additional constraint is a rationing constraint that allows us to properly define a solution concept in an environment with downward rigid wages. This constraint is a novelty in our environment and ensures that our solution concept is such that the quantity of labor that solves the households' utility-maximization problem is always equal to the quantity of labor that solves the firms' profit-maximization problem. Nevertheless, involuntary unemployment (more formally, underemployment) still exists in our environment, although it cannot be directly measured[5]

The relevance of downward rigid wages is well established in the literature. From an enthnographical point, Bewley (1995) and Campbell and Kamlani (1997) make a strong case for the claim that wages are downward rigid. Through many interviews these studies find that firms are re-

[3]A rationale for this assumption is the inflation mandate of the monetary authority. Together with downward rigid nominal wages, a stable inflation path imposes an upper bound on negative real wage changes.

[4]We say *unconstrained*, because we will introduce a rationing constraint in the households' consumption set. See the discussion below.

[5]We could measure underemployment by computing the difference between the quantity of labor under flexible wages and the quantity of labor in our environment.

luctant to cut wages unless under very severe circumstances. The main reason for this behavior is the fear of the adverse effects on morale and turnover. Empirically, there exists an extensive body of literature that suggests that wage cuts are rare in the U.S. despite studies such as Card and Hyslop (1997) finding a noticeable fraction of workers receiving wage cuts. The argument against these findings is based on the fact that they arise from reporting errors. Akerlof, Dickens, and Perry (1996) illustrate this point by collecting a small wage sample through a telephone survey in the Washington D.C. area. Altonji and Devereux (1999) and Gottschalk (2005) address the measurement error using data from the Panel Study of Income Dynamics and find no support for the claim that wages are flexible. Using data from the Employment Cost Index published by the Bureau of Labor Statistics, Lebow, Saks and Wilson (1999) reach similar conclusions. A more recent work by Hazell and Taska (2024) focuses on the wage of new hires. The authors show that wages for new vacancies are downward rigid, but upward flexible. Similar evidence exists other advanced economies. Dickens, Goette, Groshen, Holden, Messina, Schweitzer, Turunen, and Ward (2007) provide a review of international evidence based on data from 16 countries in the International Wage Flexibility Project, while Holden and Wulfsberg (2008) provide evidence from 19 OECD countries. Perhaps more important to our paper is the evidence presented by Holden and Wulfsberg (2009). Using the same sample of 19 OECD countries, the authors find that, despite there existing stronger evidence for downward nominal wage rigidity, downward real wage rigidity is statistically significant. Additionally, they find that downward real wage rigidity is more prevelant in countries with stricter employment protection legislation and higher density of union coverage, something that is also seen in Babecky, Du Caju, Kosma, Lawless, Messina, and Rõõm (2009) for European firms. All these studies are based on micro-level data. At a macro-level, Schmitt-Grohé and Uribe show that wages are downward rigid in the peripheral European countries.

This paper relates to the literature of fiscal devaluations. Adão, Correia, and Teles (2008 and 2009) show that fiscal policy can be used to implement equilibrium allocations with flexible price under sticky prices. They highlight the role of payroll subsidies to ensure that a flexible wage allocation can be implemented with constant wages. In a similar exercise, Farhi, Gopinath, and Itskhoki (2014) show that fiscal policy can be used to replicate the allocations that are obtainable with nominal devaluations. They also emphasize the role of payroll subsidies to overcome the friction imposed by wage stickiness. Our environment is a closed economy environment, but these results are still true. If we enlarge our set of fiscal instruments to include payroll taxes, the distortion imposed by downward rigid wages ceases to be relevant in the sense that it is possible to implement the flexible wage allocation. In this setting, optimal fiscal policy would be characterized by a decrease in the payroll tax rate and an increase in the labor income tax rate whenever the wage is at the lower bound and cannot fully adjust downward. By abstracting from payroll taxes, we cannot implement the flexible wage allocation. Optimal fiscal policy without payroll taxes is characterized by an increase in the labor income tax rate in whenever the wage is at the lower bound until the gap in the labor market is closed. The interpretation is that in these histories, the labor market outcome is uniquely determined by firms' demand. As a result, the government can collect higher revenue without raising additional distortions. Belchior and Reis (2024) find similar conclusions in a small open economy with downward nominal wage rigidities and a fixed exchange rate regime. More interestingly, they find that restricting free capital mobility by introducing macroprudential capital controls that reduce capital inflows during normal times is only optimal if the Ramsey planner cannot use labor income taxes in a counter-cyclical manner.

This paper also relates to the literature of the labor wedge, the gap between the marginal rate of substitution between labor and consumption and the marginal product of labor. Shimer (2009) measures the labor wedge in the United States and shows that it is counter-cyclical. Karabarbounis (2013) decomposes the labor wedge into the firm component of the labor wedge and the household component and shows that most of the variation of the labor wedge is due to the household component. The classical results in the literature of optimal Ramsey taxation suggest that the optimal labor wedge is constant. However, in our environment with downward rigid wages, the optimal labor wedge exhibits a counter-cyclical behavior. This is not surprising since the presence of downward rigid wages makes it so that whenever the current wage is at the lower bound, the marginal rate of substitution between labor and consumption is strictly less than the after-tax real wage (unless the optimal policy is implemented). Therefore, the very nature of the friction generates a counter-cyclical gap between the marginal rate of substitution between labor and consumption and the marginal product of labor. Since the optimal labor income tax is always used in a way that the marginal rate of substitution between labor and consumption is always equal to the after-tax real wage, it follows that the optimal labor wedge in our environment is counter-cyclical.

The paper is organized as follows. Section 2 introduces the environment and addresses the implications of downward rigid wages by introducing a rationing constraint on the households' consumption set. Section 3 includes the optimal taxation problem and the theoretical results about optimal income taxation policy with downward rigid wages. In section 4, we calibrate the model to match some moments of the U.S. economy and perform a numerical exercise to explore the some statistical properties of the fiscal variables. Section 5 concludes. All proofs are relegated to the Appendix.

2 Environment

Consider an infinitely-lived representative agent closed economy. Every period $t \geq 0$ a random variable s_t is drawn from the finite set $S = \{0, 1, ..., S\}$. The exogenous state of the economy at t is $s^t = (s_0, s_1, ..., s_t)$, which represents the history of realizations up until period t. Also, the probability of reaching state s^t is $\pi(s^t)$. We assume that the initial state s_0 is given. The exogenous state s^t determines aggregate productivity and government spending. Let $A(s_t)$ and $g(s_t)$ denote aggregate productivity and government spending, respectively, if the history at t is s^t.

Every (t, s^t), a continuum of measure one of identical firms transforms capital, $k(s^{t-1})$, and labor, $n(s^t)$, into a single final good using a technology represented by

$$A(s_t)F(k(s^{t-1}), n(s^t)),$$

where $F(\cdot)$ is a constant returns to scale production function, which is strictly increasing, strictly concave, twice continuously differentiable, and satisfies the Inada conditions. The final good is used for private consumption, $c(s^t)$, government spending, $g(s^t)$, and investment, $k(s^t) - (1 - \delta)k(s^{t-1})$, according to the resource constraint

$$c(s^t) + g(s_t) + k(s^t) - (1-\delta)k(s^{t-1}) = A(s_t)F(k(s^{t-1}), n(s^t)). \tag{1}$$

There is a continuum of measure one of identical households with preferences over streams of state contingent consumption and labor, $\{\{c(s^t), n(s^t)\}_{s^t}\}_{t\geq 0}$, represented by the lifetime utility function

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)U\left(c\left(s^t\right), n\left(s^t\right)\right), \tag{2}$$

where $\beta \in (0,1)$ is the subjective discount factor. The period utility function $U(\cdot,\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable with respect to consumption and strictly decreasing, strictly concave, and twice continuous differentiable with respect to labor. Additionally, it and satisfies the Inada conditions. Furthermore, we assume that preferences are standard in macroeconomics.

Assumption 1 (Standard preferences in macroeconomics)**.** The period utility function is additively separable in consumption and labor. That is, $U\left(c\left(s^t\right), n\left(s^t\right)\right) = u(c(s^t)) - \theta h(n(s^t))$, where $\theta > 0$ is the disutility of labor parameter. Moreover, it has a constant coefficient of relative risk aversion in consumption and a constant Frisch elasticity of labor supply. That is[6],

$$-\frac{u''(c(s^t))}{u'(c(s^t))}c(s^t) = \sigma \quad \text{and} \quad \frac{h''(n(s^t))}{h'(n(s^t))}n(s^t) = \psi \quad \forall (t, s^t).$$

Assumption 1 is reminiscent of the statement in Zhu (1992, Proposition 4). Consequently, his

[6] The Frisch elasticity of labor supply is $1/\psi$.

results would follow through if wages were flexible.

There is also a government that must finance a stream of exogenously given government spending, $\{\{g(s^t)\}_{s^t}\}_{t\geq 0}$, and initial debt, $b(s^0)$. We assume that the government has access to standard income taxes - a state-contingent labor income tax, $\tau_n(s^t)$, and an *ex-ante* capital income tax, $\tau_k(s^{t-1})$[7]. We further assume that the government can issue state-contingent claims, $b(s_{t+1}|s^t)$. Lastly, we assume that the government can make state-contingent transfers, $T(s^t)$, to households, but it cannot collect lump-sum taxes.

Finally, the environment is characterized by the existence of a friction in the labor market. Specifically, we impose that pre-tax wages are downward rigid. That is,

$$w(s^t) \geq \gamma w(s^{t-1}), \qquad \forall \quad (t, s^t), \tag{3}$$

where $\gamma \in [0, 1)$ measures the degree of downward wage rigidity, with $\gamma = 0$ representing the environment with flexible wages. In this model, downward wage rigidity represents a pecuniary externality because the labor market outcome at $(t-1, s^{t-1})$ affects the labor market outcomes that can be attained at all s^t. Yet, both households and firms fail to incorporate this fact when they interact in the labor market. This can cause involuntary unemployment at some history s^t if the labor market outcome at s^{t-1} has an excessively high wage $w(s^{t-1})$, making it impossible for the wage $w(s^t)$ to fully adjust downwards in response to a low realization of the aggregate productivity shock. We assume that there exists a historical wage, $w(s^{-1})$, which is exogenously given.

[7] We need to impose that the government only has access to *ex-ante* capital to ensure that the optimal capital income tax rate is uniquely defined. If the government were to use state-contingent capital income taxes, we could find many different ways of implementing the Ramsey allocation. The reason is the only thing that matters for the capital accumulation decision is the expected return on that investment. As such, there exist infinitely many different combinations of state-contingent capital income taxes that attain the expected return chosen by the Ramsey planner. Yet, there exists only one *ex-ante* capital income tax rate

2.1 The households' problem

Every (t, s^t), households receive an after-tax wage $(1 - \tau_n(s^t))w(s^t)$ for every unit of labor $n(s^t)$ and also receive an after-tax capital rent $(1 - \tau_k(s^{t-1}))r(s^t)$ for every unit of capital $k(s^{t-1})$ rented out to firms. Furthermore, households receive a payment of one unit of output from each Arrow security purchased in the previous period that pay only if s^t is realized. Finally, they receive lump-sum transfers, $T(s^t)$, from the government. Households use income to finance consumption expenditures, $c(s^t)$, capital investment expenditures, $k(s^t) - (1-\delta)k(s^{t-1})$, and purchases of state-contingent claims, $b(s_{t+1}|s^t)$, that cost $q(s_{t+1}|s^t)$ units of output in (t, s^t) and pay one unit of output in $t+1$ if s_{t+1} is realized and zero otherwise. The budget constraints are

$$
\begin{aligned}
&c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) + \sum_{s_{t+1} \in S} q(s_{t+1}|s^t) b(s_{t+1}|s^t) = \\
&\qquad (1 - \tau_n(s^t))w(s^t)n(s^t) + (1 - \tau_k(s^{t-1}))r(s^t)k(s^{t-1}) + b(s_t|s^{t-1}) + T(s^t), \quad \forall (t, s^t),
\end{aligned} \tag{4}
$$

together with a no-Ponzi games condition.

In our environment, however, we must introduce an additional restriction to the households' consumption set, which is

$$
n(s^t) \leq n^d(s^t), \quad \forall (t, s^t). \tag{5}
$$

The interpretation of this constraint is that, at any (t, s^t), the quantity of labor that households choose to work cannot exceed some value $n^d(s^t)$. Households take $n^d(s^t)$ as given, but this is an object that is determined by our solution concept and it represents the quantity of labor that firms demand at (t, s^t)[8]. The role of this constraint is to account for the fact that in states where wages cannot fully adjust downwards and (3) is binding, the quantity of labor is uniquely determined by firms. Households are off the labor supply schedule and there is involuntary unemployment.

[8] We can argue that this constraint is also part of the households' consumption set even when wages are flexible. The difference, relative to our environment, is that when wages are flexible, this constraint is never binding.

This issue is typically addressed by imposing a complementary slackness condition that states that when the the wage cannot fully adjust downwards, the quantity of labor worked is determined by firms[9]. Our additional constraint makes the complementary slackness condition a necessary condition for the solution to the utility-maximization problem. With this constraint we ensure that the quantity of labor that solves the utility-maximization problem always equals the quantity of labor that solves the profit-maximization problem. But this does not mean that there is no involuntary unemployment. In this setting, there is involuntary unemployment when (5) is binding, with the Lagrange multiplier on this constraint representing the price households are willing to pay in order to be able to work one additional marginal unit of labor.

The utility-maximization problem consists in choosing the state-contingent sequence of consumption, labor, capital and Arrow-Debreu securities that maximizes (2) subject to (4), (5), and a no-Ponzi games condition, taking the sequences of prices, taxes, lump-sum transfers, and upper bounds on labor, as well as initial conditions $k(s^{-1})$ and $b(s^{-1})$. Let $\beta^t \pi(s^t)\lambda(s^t)$ be the present-value multiplier on (5). The solution to the utility-maximization problem must satisfy the Euler equations

$$u'(c(s^t)) = \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) u'(c(s^{t+1}))[1 - \delta + (1 - \tau_k(s^t))r(s^{t+1})], \quad \forall (t, s^t) \tag{6}$$

$$u'(c(s^t)) = \beta \frac{\pi(s_{t+1}|s^t)}{q(s_{t+1}|s^t)} u'(c(s^{t+1})), \quad \forall (t, s^t), \tag{7}$$

and the intra-temporal marginal conditions and complementary slackness conditions

$$\frac{\theta h'(n(s^t))}{u'(c(s^t))} = (1 - \tau_n(s^t))w(s^t) - \frac{\lambda(s^t)}{u'(c(s^t))} \quad \forall (t, s^t) \tag{8}$$

[9] See, for example, Schmitt-Grohé and Uribe (2016).

$$\lambda(s^t)[n(s^t) - n^d(s^t)] = 0 \quad \text{and} \quad \lambda(s^t) \geq 0, \quad \forall(t, s^t). \tag{9}$$

Equations (8) and (9) are the novelty of our environment, since they result from the additional constraint on the consumption set (5). They allow for the consideration of involuntary unemployment while ensuring that the quantity of labor that solves the utility-maximization problem does not necessarily belong to the labor supply schedule. To see this, consider that there is some history s^t such that $n^d(s^t)$ is low enough. As a result, $\lambda(s^t) > 0$ and

$$\theta h'(n(s^t)) < u'(c(s^t))(1 - \tau_n(s^t))w(s^t).$$

This shows that, at the given after-tax wage, the households' disutility of one additional marginal unit of labor is less than the utility brought about $(1 - \tau_n(s^t))w(s^t)$ additional units of consumption. Consequently, households wish to increase the quantity of labor, but they cannot. Whenever (5) is binding, the solution to the utility-maximization problem is off the labor supply schedule and there is involuntary unemployment. The Lagrange multiplier $\lambda\left(s^t\right) > 0$ represents the shadow price households are willing to pay to be able to work more at the market wage rate.

2.2 The firms' problem

Firms solve a sequence of static problems. Every (t, s^t), they hire labor, $n^d(s^t)$, at the given pre-tax wage $w(s^t)$, and rent capital, $k^d(s^t)$, at the given pre-tax rent $r(s^t)$ to produce a final good with the highest possible profit. Profit is

$$A(s_t)F(k^d(s^t), n^d(s^t)) - w(s^t)n^d(s^t) - r(s^t)k^d(s^t), \quad \forall(t, s^t).$$

The solution to the profit-maximization problem satisfies the standard first-order conditions that state the equality between marginal product of an input and its real remuneration,

$$A(s_t)F_n(k^d(s^t), n^d(s^t)) = w(s^t), \quad \forall (t, s^t) \tag{10}$$

$$A(s_t)F_k(k^d(s^t), n^d(s^t)) = r(s^t), \quad \forall (t, s^t). \tag{11}$$

2.3 The government's budget constraint

Every (t, s^t), the government must finance exogenously given government spending, $g(s^t)$, debt obligations $b(s_t|s^{t-1})$, and lump-sum transfers, $T(s^t)$. To meet this obligations, the government collects state-contingent taxes on labor income and ex-ante taxes on capital income. For every unit of labor rented out to firms, the government collects a tax equal to $tau_n(s^t)w(s^t)$ and for every unit of capital rented out to firms, it collects a tax equal to $\tau_k(s^{t-1})k(s^{t-1})$. Additionally, the government can issue new state-contingent claims $b(s_{t+1}|s^t)$ that are a sold at a price $q(s_{t+1}|s^t)$ and return a payment of one unit of output in $t+1$ if s_{t+1} is realized and zero otherwise. The government's budget constraints are

$$\begin{aligned} g(s_t) + b(s_t|s^{t-1}) + T(s^t) = \tau_n(s^t)w(s^t)n(s^t) + &\tau_k(s^{t-1})r(s^t)k(s^{t-1}) + \\ &\sum_{s_{t+1} \in S} q(s_{t+1}|s^t)b(s_{t+1}|s^t), \quad \forall (t, s^t), \end{aligned} \tag{12}$$

together with a no-Ponzi games condition.

2.4 Market clearing and downward rigid wages

The market clearing conditions in the output and capital markets are standard in our environment. Respectively,

$$c(s^t) + g(s_t) + k(s^t) - (1-\delta)k(s^{t-1}) = A(s_t)F(k^d(s^t), n^d(s^t)), \quad \forall (t, s^t) \tag{13}$$

$$k^d(s^t) = k(s^{t-1}), \quad \forall (t, s^t). \tag{14}$$

The addition of constraint (5) to the households' consumption set implies that the labor market also clears. At every (t, s^t), the wage $w(s^t)$ is such that the quantity of labor that solves the utility-maximization problem equals the quantity of labor that solves the profit-maximization problem. That is,

$$n^d(s^t) = n(s^t), \quad \forall (t, s^t). \tag{15}$$

The downward wage rigidity constraint (3), when binding, plays the role of affecting the quantity of labor traded between households and firms. To understand this role, consider some history s^t where (3) is binding. Since the wage $w(s^t) = \gamma w(s^{t-1})$ cannot adjust downwards, the quantity of labor that satisfies the firms' first-order condition (10) - which defines the upper bound on the quantity of labor households can choose - implies that (5) is binding at the solution to the utility-maximization problem at history s^t. This, in turn, means that $\lambda(s^t) > 0$. A similar argument suggests that if (3) is not binding at some history $\tilde{s}^t$, then the solution to the utility-maximization problem has $\lambda(\tilde{s}^t) = 0$. We summarize this idea with the following complementary slackness condition as a requirement to our solution concept:

$$\lambda(s^t)[w(s^t) - \gamma w(s^{t-1})] = 0, \quad \text{and} \quad \lambda(s^t) \geq 0, \quad \forall (t, s^t). \tag{16}$$

2.5 Solution concept with downward rigid wages

We finish this section with a definition of our solution concept in the presence of downward rigid wages. We refer to this solution concept as an *equilibrium with downward rigid wages*.

Definition 1 (**Equilibrium with downward rigid wages).** An *equilibrium with downward rigid wages* is an allocation for households $\{\{c(s^t), n(s^t), k(s^t), b(s_{t+1}|s^t)\}_{s^t}\}_{t\geq 0}$, an allocation for firms

$\{\{k^d(s^t), n^d(s^t)\}_{s^t}\}_{t\geq 0}$, prices $\{\{w(s^t), r(s^t), q(s_{t+1}|s^t)\}_{s^t}\}_{t\geq 0}$, Lagrange multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and policies $\{\{\tau_n(s^t), \tau_k(s^t), T(s^t)\}_{s^t}\}_{t\geq 0}$, such that, given the sequences $\{\{A(s^t, g(s^t)\}_{s^t}\}_{t\geq 0}$ and initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$, (i) the household allocation solves the utility-maximization problem, (ii) the firm allocation solves the profit maximization problem, (iii) the government's budget constraints are satisfied, (iv) markets clear, and (v), wages and multipliers satisfy the complementary slackness conditions.

Following definition (1), an equilibrium with downward rigid wages is characterized by conditions (3) - (16). The next section formulates the optimal taxation problem with downward rigid wages and characterizes the optimal fiscal policy.

3 Optimal Taxes

In order to characterize the optimal Ramsey income taxes, we start by characterizing the set of attainable allocations. Our approach follows Lucas and Stokey (1983).

Proposition 1 (Set of attainable allocations). An allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$ can be attained as an *equilibrium with downward rigid wages* with exogenously given stochastic sequences of aggregate productivity and government spending $\left\{\left\{A\left(s^t\right), g\left(s^t\right)\right\}_{s^t}\right\}_{t\geq 0}$ and initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$ if and only if it satisfies the resource constraints (1) for all (t, s^t), the implementability condition

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[u'(c(s^t))c(s^t) - \theta h'(n(s^t))n(s^t)] \geq \mathcal{W}_0, \tag{17}$$

where

$$\mathcal{W}_0 = u'(c(s^0))\{[(1-\delta+(1-\tau_k(s^{-1}))F_k(k(s^{-1}), n(s^0))]k(s^{-1}) + b(s^{-1})\},$$

and the following constraints on the marginal product of labor

$$A(s^t)F_n(k(s^{t-1}),n(s^t)) \geq \gamma A(s^{t-1})F_n(k(s^{t-2}),n(s^t)), \quad \forall (t \geq 1, s^t)$$
$$\text{and} \quad A(s^0)F_n(k(s^{-1}),n(s^0)) \geq \gamma w(s^{-1}) \quad \text{for} \quad t = 0. \tag{18}$$

Proof. See Appendix A. ■

Proposition (1) shows that an attainable allocation is characterized by the resource constraints (1) and the implementability condition (17). These are the same conditions we obtain in an environment with flexible wages. But wages are downward rigid in our environment and a benevolent social planner takes the pecuniary externality into consideration. That is, a benevolent social planner takes into account that the social marginal cost of a higher wage at some history s^t, $w(s^t)$, includes the fact that the wage at some history s^{t+1} that follows from s^t may not be able to fully adjust downwards. This is why the constraints on the marginal product of labor (18) are part of the characterization of attainable allocations.

The Ramsey allocation - the best allocation in the set of attainable allocations - is the allocation that maximizes (2) subject to conditions (1), (17), and (18), given the initial stock of capital $k\left(s^{-1}\right)$ and historical wage $w\left(s^{-1}\right)$. Additionally, we follow Armenter (2008) and Chari, Nicolini, and Teles (2020) and assume that initial wealth in utility terms, $\mathcal{W}_0$. The interpretation is that the government cannot confiscate initial wealth, neither directly nor indirectly. This assumption implies that all periods look alike, which simplifies our recursive representation of the problem (see section 4). In an environment with flexible wages, an exogenously given initial wealth in marginal utility terms ensures that the Ramsey allocation never has inter-temporal distortions, i.e., the optimal *ex-ante* capital income tax equals zero forever[10]. This assumption also ensures that the Ramsey allocation with flexible wages has the same intra-temporal wedge every period. The proposition that follows shows that these results do not hold when wages are downward rigid.

Proposition 2 (Optimal Ramsey taxes). Let $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$ be a Ramsey allocation with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}), \mathcal{W}_0)$. The state-contingent labor income taxes

[10] This result depends on the assumption that preferences are standard preferences in macroeconomics, as they are in our environment per Assumption 1.

and ex-ante capital income taxes that implement the Ramsey allocation are, respectively,

$$\tau_n(s^t) = 1 - \frac{1+\mu(1-\sigma)}{1+\mu(1+\psi)} + \beta\gamma \frac{F_{nn}(k(s^{t-1}), n(s^t))}{F_n(k(s^{t-1}), n(s^t))} \frac{\mathbb{E}_t[\eta(s^{t+1})]}{u'(c(s^t))[1+\mu(1+\psi)]} - \frac{F_{nn}(k(s^{t-1}), n(s^t))}{F_n(k(s^{t-1}), n(s^t))} \frac{\eta(s^t)}{u'(c(s^t))[1+\mu(1+\psi)]} \quad \forall (t, s^t) \tag{19}$$

and

$$\tau_k(s^t) = \frac{\beta\gamma \mathbb{E}_t[A(s^{t+1}) F_{nk}(k(s^t), n(s^{t+1})) \mathbb{E}_{t+1}[\eta(s^{t+2})]]}{[1+\mu(1-\sigma)] \mathbb{E}_t[u'(c(s^{t+1})) A(s^{t+1}) F_k(k(s^t), n(s^{t+1}))} - \frac{\mathbb{E}_t[A(s^{t+1}) F_{nk}(k(s^t), n(s^{t+1})) \eta(s^{t+1})]}{[1+\mu(1-\sigma)] \mathbb{E}_t[u'(c(s^{t+1})) A(s^{t+1}) F_k(k(s^t), n(s^{t+1}))} \quad \forall (t, s^t), \tag{20}$$

where $\mu > 0$ is the Lagrange multiplier of the implementability condition (17), and $\beta^t \pi(s^t) \eta(s^t) \geq 0$ is the present value Lagrange multiplier of the constraint on the marginal product of labor (18) at history s^t.

Proof. See Appendix B. ■

Proposition (2) shows how downward rigid wages affect the optimal state-contingent labor income tax and the optimal ex-ante capital income tax. The optimal state-contingent labor income tax (19) is not constant over the business cycle, even under the assumption that preferences are standard preferences in macroeconomics. To see how the optimal labor income tax moves over the business cycle, consider some history s^t such that the Ramsey allocation has $\eta(s^t) = 0$. The optimal labor income tax is

$$\tau_n(s^t) = 1 - \frac{1+\mu(1-\sigma)}{1+\mu(1+\psi)} + \beta\gamma \frac{F_{nn}(k(s^{t-1}), n(s^t))}{F_n(k(s^{t-1}), n(s^t))} \frac{\mathbb{E}_t[\eta(s^{t+1})]}{u'(c(s^t))[1+\mu(1+\psi)]}.$$

The second term in this equation exists because of the nature of the wage rigidity. The Ramsey planner takes into consideration that the choice for the marginal product of labor at history s^t affects the possible choices for the marginal product of labor for all histories s^{t+1} that follow from

s^t. Since the wage is above the lower bound at history s^t, optimal policy consists in using the labor income tax to induce a lower equilibrium wage $w(s^t)$. This requires giving households an incentive to increase the quantity of labor they want to work with a lower labor income tax rate[11]. By reducing the equilibrium wage at history s^t, the benevolent social planner is effectively relaxing the constraints on the marginal product of labor (18) for all histories s^{t+1} that follow from s^t. If, on the other hand, history s^t is such the Ramsey allocation has $\eta(s^t) > 0$, this mechanism is inaccessible because the wage is at the lower bound and cannot adjust downwards. In this case, the labor market outcome is determined by the firms' demand (10). Despite choosing the quantity of labor that solves the profit-maximization problem, households are willing to sacrifice some resources to be able to work a little more. That is, the constraint (5) is binding. Optimal policy consists in setting a higher labor income tax rate to the point where the constraint (5) is no longer binding at the solution to the utility-maximization problem at history s^t. This is because a higher labor income tax rate when the wage is at the lower bound does not change the labor market outcome and, as such, results in an increase in revenue without raising additional distortions.

The optimal *ex-ante* capital income tax (20) is not necessarily equal to zero over the business cycle. To see how the optimal capital income tax moves over the business cycle, first observe that it only depends on expectations about the future. The reason is that the set of fiscal instruments includes only *ex-ante* capital income taxes and we restrict of fiscal instruments in this manner to ensure that the optimal capital income tax rate is uniquely determined. Now, consider some history s^t where $\eta(s^{t+1}) > 0$ for some history s^{t+1} that follows from s^t. This means that the constraint on the marginal product of labor (18) is binding at some history s^{t+1} that follows from s^t. Holding everything else constant, optimal policy consists in decreasing the capital income tax rate that is to be collected at s^{t+1}. The interpretation is that a lower *ex-ante* capital income tax increases investment at s^t, leading to a higher stock of capital at all histories s^{t+1} that follow from s^t. Since the marginal product of labor depends positively on the stock of capital, this policy results in a higher marginal product of capital at all histories s^{t+1}, relaxing the downward wage rigidity constraints. This idea

[11]Recall that the production function $F(\cdot)$ is strictly increasing and strictly concave in both its arguments. That is, $F_{nn}(\cdot) < 0$.

is captured in the second line of (20). Next, consider that the Ramsey allocation has $\eta(s^{t+2}) > 0$ for some history s^{t+2} that follows from s^t, meaning that the constraint on the marginal production of labor (18) is binding at some history s^{t+2} that follows from s^t. In this case, optimal policy consists in increasing the capital income tax rate collected at all histories s^{t+1} that follow from s^t. This has to do with the fact that the lower bound on the wage at all histories s^{t+2} depends on the realized wage $w(s^{t+1})$. A higher capital income tax rate $\tau_k(s^t)$ reduces investment and leads to a lower stock of capital at all histories s^{t+1} that follow from s^t. Everything else constant, a lower stock of capital results in a lower equilibrium wage $w(s^{t+1})$, which effectively relaxes the downward wage rigidity constraints (3) for all histories s^{t+2} that follow from s^t. This idea is captured by the first line of (20). Finally, observe that this analysis depends on the assumption that the production function is a standard neoclassical production function with $F_{nk}(\cdot) > 0$. If labor and capital were not related in production, i.e., $F_{nk}(\cdot) = 0$, the optimal ex-ante capital income tax would be zero for all (t, s^t).

In the next section, we solve the Ramsey problem numerically for a model calibrated to the U.S. economy to see the mechanisms described in (19) and (20) and discussed above. Through this quantitative exercise, we will be able to fully explore the cyclical behavior of the optimal Ramsey taxes.

4 Quantitative Results

We start this section by formulating the Ramsey problem recursively. This is a relatively straightforward exercise, but we want to emphasize some particular aspects of the formulation as well as the role of some of our assumptions. We then explain our calibration strategy and introduce our quantitative results.

4.1 Recursive formulation of the Ramsey problem

Denote by $\mathcal{W}(s^t)$ the value of wealth in utility terms at history s^t. This means that

$$\mathcal{W}(s^t) = u'(c(s^t))\{[1-\delta+(1-\tau_k(s^{t-1}))F_k(k(s^{t-1}),n(s^t))]k(s^{t-1})+b(s_t|s^{t-1})\}.$$

We can use this variable to write the intra-temporal implementability constraints in a recursive fashion:

$$u'(c(s^t))c(s^t) - \theta h'(n(s^t))n(s^t) + \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)\mathcal{W}(s_{t+1}|s^t) = \mathcal{W}(s^t) + \lambda(s^t)n(s^t) + u'(c(s^t))T(s^t) \quad \forall(t,s^t). \tag{21}$$

This allows us to treat the value of wealth in marginal utility terms as a state variable. As explained in Chari, Nicolini, and Teles (2020), promises of wealth in utility terms constrain the policy choices of the Ramsey planner. Furthermore, our assumption that initial wealth in marginal utility terms, $\mathcal{W}_0$, is exogenous and the consideration that the government has access to lump-sum transfers at all (t, s^t) make all periods look alike. This is a simplifying assumption that eliminates the difference between period zero and all future periods, which is standard in the literature of optimal Ramsey taxation. Therefore, the recursive formulation of our Ramsey problem consists of one single Bellman equation.

The constraint on the marginal product of labor (18) suggests that we must keep track of previous shock, stock of capital, and labor. However, we find it easier to keep track of the realization of the previous wage, since it includes all this information. We are effectively adding an additional sequence of choice variables to the Ramsey problem, $\{\{w(s^t)\}_{s^t}\}_{t\geq 0}$, as well as an additional sequence of constraints,

$$w(s^t) = A(s^t)F_n(k(s^t),n(s^t)) \quad \forall(t,s^t). \tag{22}$$

It is straightforward to see that the Ramsey allocation $\{\{c(s^t),n(s^t),k(s^t)\}_{s^t}\}_{t\geq 0}$ also solves the Ramsey problem augmented by the introduction of these additional constraints, which consists in choosing the stochastic sequence $\left\{\left\{c\left(s^t\right), n\left(s^t\right), k\left(s^t\right), w\left(s^t\right), \mathcal{W}\left(s^{t+1}\right)\right\}_{s^t}\right\}_{t\geq 0}$ that maximizes (2) subject to (1), (3), (21), (22), and the transversality condition

$$\lim_{t\longrightarrow\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u'\left(c\left(s^t\right)\right)\mathcal{W}\left(s^t\right)=0.$$

The Ramsey planner enters every period with a predetermined stock of capital, k, the previous realization of the wage, w_{-1}, and a vector of wealth in utility terms, $\{\mathcal{W}(s)\}_{s\in S}$. Before making any decisions, it observes the realization of the random variable $s\in S$, which determines the value of aggregate productivity, $A(s)$, government expenditures, $g(s)$, and the wealth in marginal utility terms $\mathcal{W}(s)$ that must be delivered this period. This means that the state vector is $(s,k,w_{-1},\mathcal{W}(s))$. After observing the realization of the exogenous state s, the Ramsey planner chooses consumption c, labor n, wage w, the stock of capital for next period k', and the vector of promised wealth in marginal utility terms $\{\mathcal{W}'(s')\}_{s'\in S}$. Let $V(s,k,w_{-1},\mathcal{W}(s))$ denote the value associated with the Ramsey plan $(c,n,k',w,\{\mathcal{W}'(s')_{s'\in S})$ at state $(s,k,w_{-1},\mathcal{W}(s))$. This value satisfies the Bellman equation[12]

$$\begin{aligned} V(s,k,w_{-1},\mathcal{W}) = &\max_{(c,n,k',w,\{\mathcal{W}'(s')_{s'\in S})} u(c)-\theta h(n)+\beta\sum_{s'\in S}\pi(s'|s)V(s',k',w,\mathcal{W}'(s')) \\ \text{s.t.}\quad & c+g(s)+k'-(1-\delta)k=A(s)F(k,n) \\ & u'(c)c-\theta h'(n)n+\beta\sum_{s'\in S}\pi(s'|s)\mathcal{W}'(s')=\mathcal{W} \\ & w=A(s)F_n(k,n) \\ & w\geq\gamma w_{-1}. \end{aligned}$$

4.2 Functional Forms, Parameters and Calibration

The functional forms chosen for our quantitative exercise are as follows. The utility function is

[12] The assumption that $\mathcal{W}_0$ is large enough so that the the implementability condition (17) is binding at the solution to the Ramsey problem implies that the Ramsey allocation has $\lambda(s^t)=T(s^t)=0$ for all (t,s^t). We simplify the Bellman equation by imposing this result and removing the Lagrange multiplier and lump-sum transfers from the implementability constraint.

$$u(c) - \theta h(n) = \frac{c^{1-\sigma}}{1-\sigma} - \theta \frac{n^{1+\psi}}{1+\psi},$$

where σ represents the inter-temporal elasticity of substitution for consumption, and ψ represents the inverse of the Frisch elasticity of labor supply. With preferences represented by such an utility function, the traditional results of uniform labor income taxation and zero capital income taxation over the business cycle hold. The production function is a classic neoclassical production function of the Cobb-Douglas type,

$$F(k, n) = k^{\alpha} n^{1-\alpha},$$

where α denotes the capital income share.

For our calibration exercise, we consider a simple version of the U.S. economy corresponding to a deterministic version of our model and calibrate the parameters following the standard procedure in the literature. We set the discount factor $\beta = 0.98$, which implies that the real interest rate is approximately 2%, and we set the inter-temporal elasticity of substitution for consumption $\sigma = 2$. These are typical values for these parameters. We follow Boar and Midrigan (2021) and set the disutility of labor parameter $\theta = 1$ and the Frisch elasticity of labor supply $\psi = 2$. We follow Chari, Christiano, and Kehoe (1994) and set the capital share of output $\alpha = 0.34$, and the depreciation rate $\delta = 0.08$. Furthermore, we take the degree of downward wage rigidity $\gamma = 0.99$ from Schmitt-Grohé and Uribe (2016) and we normalize aggregate productivity to $A = 1$. Table 1 summarizes this information.

We assume that in the initial steady-state, government consumption is 20% of output and that the steady-steady level of government debt is 70% of output[13] Also, we follow Barro and Furman (2018) and consider that the capital income tax rate is 38%. In our environment, the capital income

[13]Both these values are consistent with data from the U.S. economy. (See the NIPA table 1.1.5 for the ratio of government spending to output and series for total public debt as a percentage of gross domestic product from the Federal Reserve Bank of St. Louis.)

Table 1
Externally Calibrated Parameters

Parameter	Description	Value	Source
β	subjective discount factor	0.98	Standard
σ	inter-temporal elasticity of substitution for c	2.00	Standard
ψ	inverse of Frisch elasticity of labor supply	2.00	Boar and Midrigan (2022)
θ	disutility of labor	1.00	Boar and Midrigan (2022)
α	capital income share	0.34	CCK (1994)
δ	depreciation rate	0.08	CCK (1994)
γ	degree of downward wage rigidity	0.99	Schmitt-Grohé and Uribe (2016)
A	aggregate productivity	1.00	Normalization

tax rate is applied to rental rate of capital before depreciation whereas only capital income net of depreciation is subject to the tax rate in Barro and Furman (2018). Nevertheless, there is an equivalent capital income tax rate in our environment, which is given by $\tau_k = \hat{\tau}_k \left(1 - \frac{\delta}{\alpha}\frac{k}{y}\right)$. These are our calibration targets, which we summarize in Table 2.

We set the labor income tax rate so that the government budget constraint in this initial steady-state is satisfied. It follows from our calibration exercise that the labor income tax rate associated with this initial steady-state is 26.76%. Our procedure also determines the deterministic level of government consumption, which is 0.3414, and a capital income tax rate equal to 11.08%. Table 3 summarizes the results of our calibration procedure. We look at the value of capital-to-output

Table 2
Calibration Targets

Parameter	Description	Target	Source
g/y	ratio of government spending to output	0.20	NIPA Table 1.1.5
b/y	ratio of public debt to output	0.7	FRED
τ_k	tax rate on capital income	0.38	Barro and Furman (2018, Table 4)

Table 3
Internally Calibrated Parameters

Parameter	Description	Value
g	government spending	0.34
τ_n	labor income tax rate	0.2676
τ_k	tax rate on capital income	0.1108

ratio induced by our calibration to confirm the validity of our results. Data on the U.S. economy suggests that capital-to-output ratio is around 3 (see NIPA table 1.1.5 for data on Gross Domestic Product and FAA table 1.1 for data on capital stock). Our induced level of capital-to-output ratio is 3.01.

We borrow the processes for aggregate productivity and government spending from Chari, Christiano, and Kehoe (1991). We let the productivity shock z be a zero mean symmetric two-state Markov chain with states z_L and z_H and transition probabilities $\Pr(z_{t+1} = z_i | z_t = z_i) = \pi$ for $i = L, H$. Then, aggregate productivity A takes values from the set $\{1 - z_L, 1 + z_H\}$. Similarly, we let the government spending shock $\tilde{g}$ be a zero mean symmetric two-state Markov chain with states $\tilde{g}_L$ and $\tilde{g}_H$ and transition probabilities $\Pr(\tilde{g}_{t+1} = \tilde{g}_i | \tilde{g}_t = \tilde{g}_i) = \phi$ for $i = L, H$. Then, government spending g takes values from the set $\{0.3414 - \tilde{g}_L, 0.3414 + \tilde{g}_H\}$. Table 4 presents the parameter values for these stochastic processes.[14]

Table 4
Parameter Values for Markov Chains for Government Spending Shock

	Parameters and Values
Technology shock	$z_L = 0.04$ $z_H = 0.04$ $\pi = 0.91$
Government spending shock	$\tilde{g}_L = 0.0235$ $\tilde{g}_H = 0.0235$ $\phi = 0.95$

Notes: The values for z_L, z_H, π, and ϕ are taken directly from Chari, Christiano, and Kehoe (1991). The values for $\tilde{g}_L$ and $\tilde{g}_H$ were chosen so that the government spending shock is the same fraction of the steady-state value of government spending as in Chari, Christiano, and Kehoe (1991).

The steady-state allocations induced by our calibration induce an initial wealth measured in marginal utility terms equal to $\omega_0 = 7.1$. At period zero, aggregate productivity and government consump-

[14] The quantitative results presented in section 4.3 and section 4.4 below are still preliminary and refer to a version of the model without the government spending shock and in which the transition probability for the aggregate productivity shock is $\pi = 0.8$ instead.

tion fall below average and follow their respective independent Markov chains thereafter. The government switches to the Ramsey policy in period zero that delivers wealth in marginal utility terms in period zero equal to $\omega_{ss} = 7.1$ and takes into account the fact that wages are downward rigid. The initial capital stock is $k_{ss} = 5.14$ and the historical wage that defines the lower bound on wages in period zero is $w_{ss} = 1.16$.

4.3 Cyclical properties of the optimal Ramsey taxes

We solve the functional equation problem that defines the Ramsey problem in recursive fashion and obtain policy functions for consumption, labor, capital stock, and wages. Subsequently, we simulate a time series with 5,000 periods starting from the deterministic steady state discussed above and drop the first 1,000 periods to ensure that we take the fiscal variables from their stationary distributions. We compute the mean, standard deviation, and correlation coefficient of both the labor income tax and capital income tax that implement the Ramsey allocation. Table 5 reports the results. The first column shows the results of the model with flexible wages. This is the model with $\gamma = 0$. Given our assumptions on preferences, it is not surprising that the standard deviation and correlation with output of the optimal labor and capital income taxes are exactly equal to zero. The results for our baseline model with $\gamma = 0.99$ are in the second column. We can see that, on average, the optimal labor income tax is 1 percentage point higher than under flexible wages. More interestingly, the standard deviation is now 15.77%, because the Ramsey planner no longer finds it optimal to uniformly tax labor income. Instead, it wants to use the labor income tax to collect revenue without raising additional distortions when the current wage is at the lower bound; otherwise it wants to use to reduce the current wage and the probability that the lower bound binds in the future. As a result, the optimal labor income behaves counter-cyclically to output with a correlation coefficient of -0.95. The optimal *ex-ante* capital income tax rate in the baseline model is, on average, negative and equal to -1.40%, and it displays a standard deviation equal to 27.02%. The higher volatility of the optimal capital income tax can be attributed to the fact that it depends only on expectations about the future. The most interesting result has to do with the co-movement with respect to output. Our results suggest that the optimal capital income tax is highly pro-cyclical. We suspect that this follows from the high persistence of the productivity shock. If a negative productivity shock is expected in the following period, the Ramsey planner finds it optimal to subsidize capital income to reduce exposure to the lower bound. Once the negative shock hits, the Ramsey planner sees it very likely that the following shock will also be negative, and, once again, it wants to subsidize capital income[15].

It is instructive to see time series for our relevant variables to see how downward rigid wages drive our conclusions regarding the optimal linear income taxes. Figure 1 shows a portion of our simulated time series for wages. We include the time series for the endogenously determined lower bound to clearly show when the wages hit the lower bound. The shaded grey lines represent periods in which the productivity shock is negative and productivity falls below average. In the baseline model, periods in which the productivity shock is negative correspond to periods in which the wage hits the lower bound. This need not be the case always, but a high degree of downward rigid wages $\gamma = 0.99$ makes it more likely.

Figure 2 plots the the simulated time series for output (left vertical axis) and the simulated time series for the optimal labor income tax rate (right vertical axis). In the previous section, we discussed the two opposite forces that govern the dynamics of the optimal state-contingent labor income tax (19). It is very easy to see that these forces imply that the optimal labor income tax behaves counter-cyclically to output. The nature of the wage rigidity – the fact that lower bound on the wage is endogenous – makes it optimal to have a lower labor income tax rate whenever the wage is above the lower bound. The rationale is that lower labor income tax rate gives incentives for households to supply more hours of labor for each and every wage, resulting in a lower equilibrium wage at (t, s^t). Consequently, the probability that the endogenously determined price floor is binding at some state s^{t+1} is lower. This mechanism displays how the optimal labor income tax rate is used to internalize the pecuniary externality. However, in periods where the wage is at the lower bound and cannot fully adjust downwards, it is optimal to have a labor income tax

[15] Our hypothesis is that the more *iid* the productivity shock is, the more acyclical the capital income tax is. We will this hypothesis by simulating the model with different levels of the transition probability π.

Table 5
Properties of Tax Rates

Tax Rates	Flexible Wages Model ($\gamma = 0$)	Baseline Downward Rigid Wages Model ($\gamma = 0.99$)	Downward Rigid Wages Model ($\gamma = 0.98$)	Downward Rigid Wages Model ($\gamma = 0.97$)
Labor Income Tax				
Mean	30.71	31.72	31.27	31.05
Standard deviation	0.00	15.77	8.09	5.32
Correlation with output	0.00	-0.95	-0.85	-0.75
Capital Income Tax				
Mean	0.00	-1.40	-0.48	-0.15
Standard deviation	0.00	27.02	9.71	3.52
Correlation with output	0.00	0.90	0.65	0.27

Notes: All statistics are obtained from simulating a realization of 5,000 periods and then dropping the first 1,000 periods. The means and standard deviations are in percentage terms.

The values in this table are still preliminary. They assume the steady-state value of government consumption is $g = 0.21$ and that there is only the technology shock with transition probability $\pi = 0.80$.

high enough so that the rationing constraint (5) is not binding at the solution to the utility-maximization problem. In doing so, the government explores the opportunity to collect revenue without introducing additional distortions. In our environment, the optimal labor income tax ensures that the constraint (5) is never binding at the solution to the utility-maximization problem, not even in states where the the wage is at its lower bound. That is, the optimal labor income tax ensures that $\lambda(s^t) = 0$ for all (t, s^t). This does not mean, however, that optimal policy eliminates the effect of downward rigid wages. If that were true, the allocation would be the same as under flexible wages and this is not the case. Indeed, when the downward wage rigidity constraint (3) is binding, the labor market outcome is characterized by a quantity of labor below the flexible-wage counterpart. The reason is that the government does not have access to a fiscal instrument that can reduce the effective cost of labor to firms and increase the quantity of labor demanded at the market wage rate. An example of such a fiscal instrument is a state-contingent payroll tax and we can show that a combination of lower payroll taxes and higher labor income taxes would eliminate the effects of downward rigid wages[16]. Moreover, the implication of optimal policy in our environment is that the intra-temporal marginal conditions (8) become

$$\frac{\theta h'(n(s^t))}{u'(c(s^t))} = (1 - \tau_n(s^t))w(s^t) \quad \forall (t, s^t).$$

Under the optimal labor income tax policy, the intra-temporal wedge is akin to the definition found in the literature of business cycle accounting – the ratio of the marginal rate of substitution between labor and consumption to the marginal product of labor. Our results, although normative, are not unrealistic. Shimer (2009), for example, computes the labor wedge for the United States and concludes that it is counter-cyclical.

In section 3, we also discussed the two opposite forces that govern the dynamics of the optimal

[16]With payroll taxes, the wage $w(s^t)$ is no longer the relevant cost of labor to firms. Instead, the effective cost of labor is $(1 + \tau_p(s^t))w(s^t)$, where $\tau_p(s^t)$ is the payroll tax rate at history s^t. Payroll taxes allow the government to control the first-order condition (10), which makes it possible to induce firms to hire any arbitrary quantity of labor, even when the downward wage rigidity constraint (3) is binding. See Adão, Correia, and Teles (2008, 2009) and Farhi, Gopinath, and Itskhoki (2014) for a detailed discussion of how fiscal policy can be used to implement flexible-price allocations when prices are sticky.

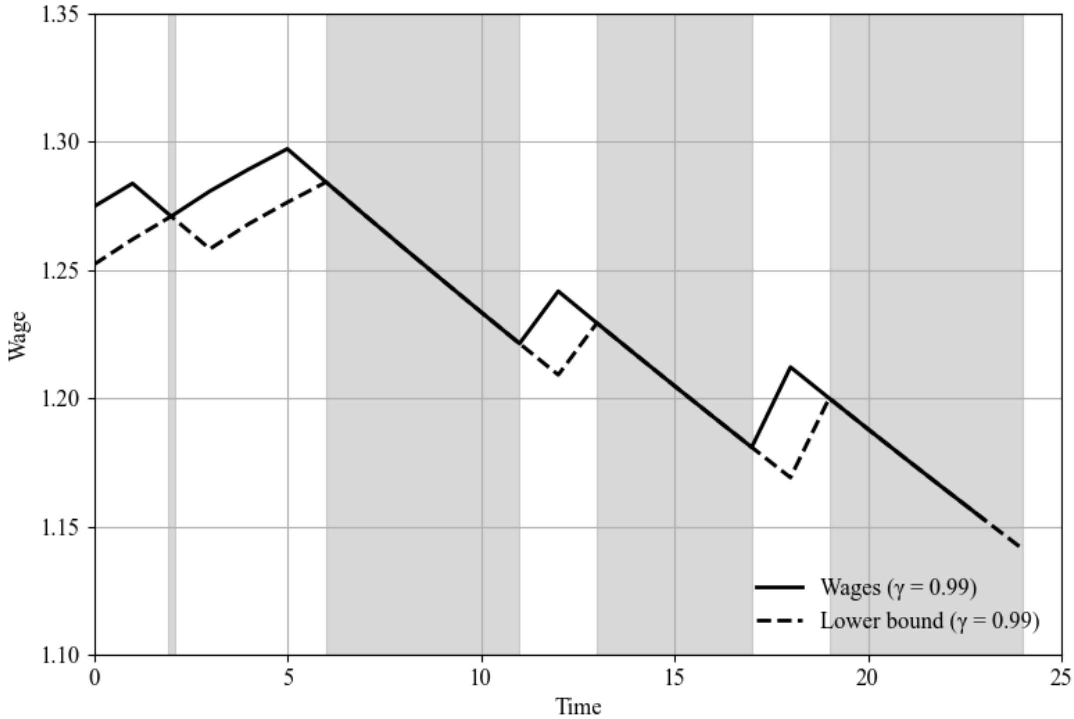

Figure 1: Time series for wages and lower bounds

Notes: We solve the Bellman equation for the Ramsey problem and simulate a time series for the aggregate productivity shock with 5,000 periods. The figure presents the 25 periods within the interval 3,000 - 3,024 to ensure that the variables are obtained from their stationary distributions. The time series for the wage is derived from the policy functions for the wage. The grey bars show the periods in which productivity is below average.

ex-ante capital income tax (20). Figure 3 displays these properties by plotting the simulated time series for both output (right vertical axis) and the optimal ex-ante capital income tax (left vertical axis). Once again, the shaded grey bars represent periods in which the productivity shock is negative and the wage is at the lower bound. We can see that the optimal *ex-ante* capital income tax exhibits a pro-cyclical behavior. A period in which the wage is at the lower bound and output is lower was assigned a positive probability in the previous period when the capital income tax rate was set. In response to this positive probability event the Ramsey planner finds it optimal to subsidize capital income. The interpretation of this result is as discussed above. A lower ex-ante capital income tax leads to a higher marginal product of labor when the productivity shock is low, thus making the effects of downward rigid wages less severe. On the other hand, a history s^t in

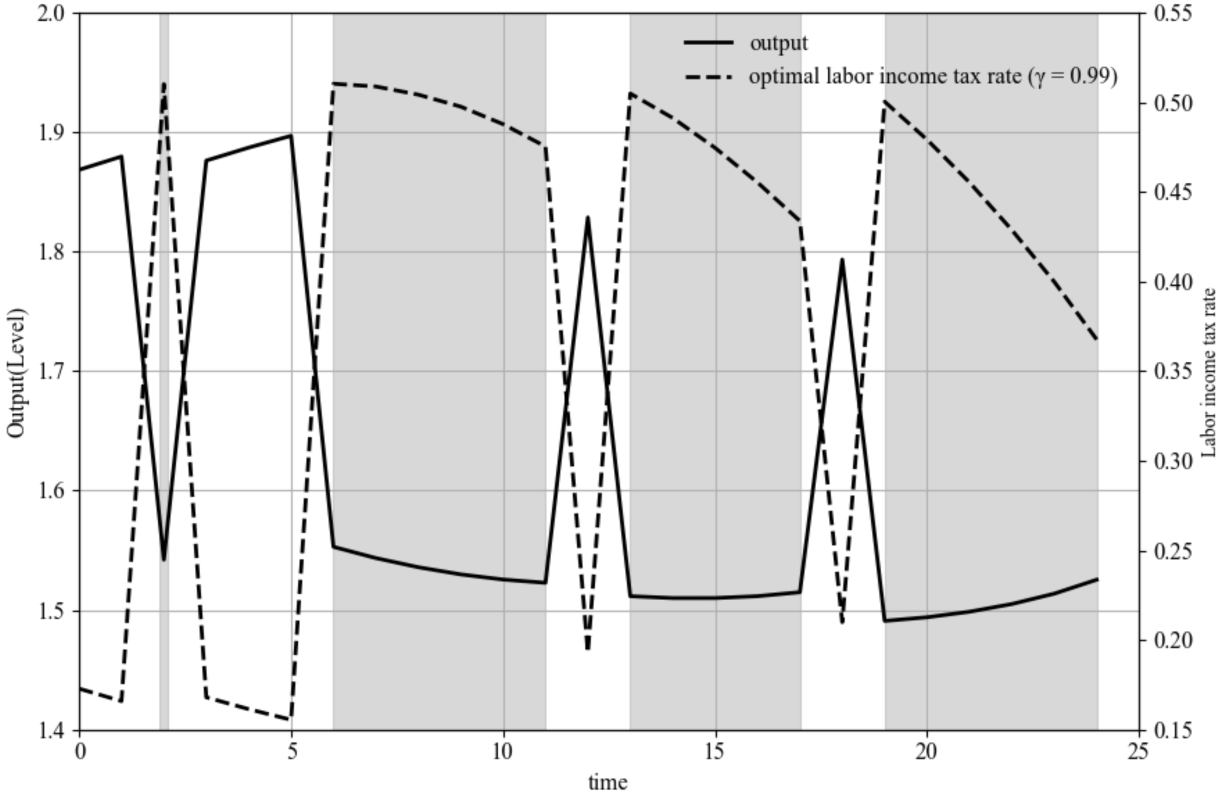

Figure 2: Counter-cyclical behavior of optimal labor income tax

Notes: We solve the Bellman equation for the Ramsey problem and simulate a time series for the aggregate productivity shock with 5,000 periods. The figure presents the 25 periods within the interval 3,000 - 3,024 to ensure that the variables are obtained from their stationary distributions. The left vertical axis displays output levels and the right vertical axis displays labor income tax rates. The time series for output is derived from the policy functions for capital and labor. To obtain the time series for the labor income tax rate, we use equation (8) with $\lambda(s^t) = 0$. The grey bars show the periods in which productivity is below average.

which the wage is above the lower bound and output is higher is a period in which the Ramsey planner finds it optimal to have a higher capital income tax rate $\tau_k\left(s^{t-1}\right)$. This follows from the attempt to internalize the effects of the pecuniary externality. A higher capital income tax set at s^{t-1} reduces the stock of capital in the period in which the capital income tax is collected $k\left(s^{t-1}\right)$. Consequently, the wage is low in this period, leading to a low floor on the wage at all s^{t+1} that follow from s^{t-1}.

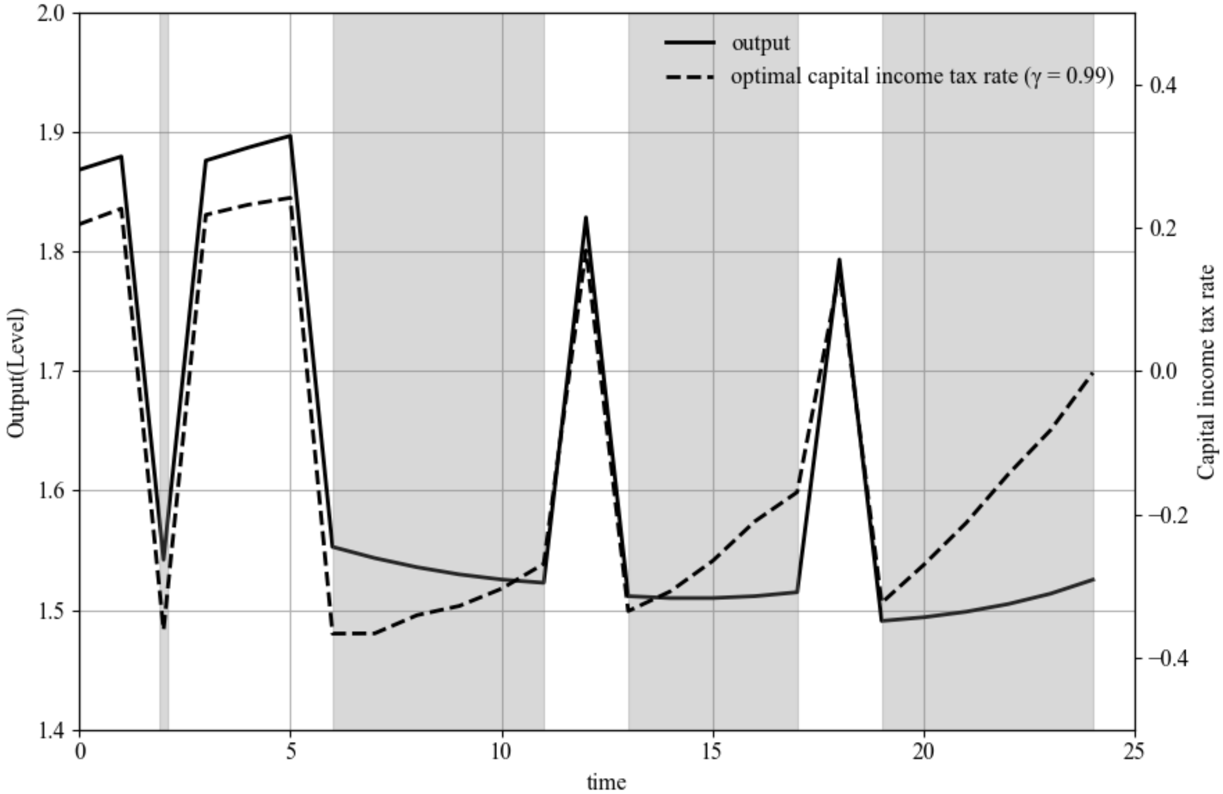

Figure 3: Pro-cyclical behavior of optimal capital income tax

Notes: We solve the Bellman equation for the Ramsey problem and simulate a time series for the aggregate productivity shock with 5,000 periods. The figure presents the 25 periods within the interval 3,000 - 3,024 to ensure that the variables are obtained from their stationary distributions. The left vertical axis displays output levels and the right vertical axis displays capital income tax rates. The time series for output is derived from the policy functions for capital and labor. To obtain the time series for the ex-ante capital income tax rate, we use the Euler equations (6). The grey bars show the periods in which productivity is below average.

4.4 Sensitivity Analysis

Even though the empirical literature on downward rigid wages is extensive and most authors have converged to the idea that wages are downward rigid, we find no consensus regarding the value that γ should take. This parameter determines the intensity of downward rigid wages and, as such, it is crucial is for the quantitative implications of downward rigid wages on optimal fiscal policy. For this reason, we also solve the Ramsey problem for different values of this parameter. Table 5 above reports the properties of the fiscal variables for versions of the model with $\gamma = 0.98$ and $\gamma = 0.97$.

As one would expect, all the properties of both the labor income tax rate and the capital income tax rate presented in Table 5 – mean, standard deviation, and correlation with output – decrease as γ decreases. Lower values of the intensity of downward rigid wages γ imply that the degree to which wages can adjust downward between consecutive periods is larger. In other words, an economy with a lower γ is closer to the version of the economy with flexible wages, where uniform labor income taxation and zero capital income taxes are optimal. Therefore, the properties of the fiscal variables are successively closer to the values they take in an environment with flexible wages as γ decreases from in the interval $\{0.97, 0.98, 0.99\}$. Naturally, this implies that the costs of downward rigid wages decrease as γ decreases. Nevertheless, the optimal labor income tax rate is still strongly counter-cyclical, with the correlation coefficient with output being -0.75 when $\gamma = 0.97$. The optimal capital income tax rate does not co-move with output as strongly, although it is still pro-cyclical. When $\gamma = 0.97$, which is the lowest value we considered, the correlation coefficient with output is 0.27.

5 Conclusion

In this paper, we study the implications on optimal income taxation of downward rigid wages. We focus on a standard neoclassical economy and impose a friction in the labor market that makes it impossible for the real wage to fall below some endogenously determined lower bound. The endogeneity of this lower bound introduces a pecuniary externality since private agents fail to recognize the effect that their current decisions have on the lower bound in the future. In this environment, the classical results in the literature of optimal taxation no longer hold. In particular, the optimal state-contingent labor income tax rate is not constant over time and the optimal ex-ante capital income tax is not equal to zero in every period, even if preferences are standard preferences in macroeconomics. The reason is that these taxes can be used to alleviate the effects of downward rigid wages, although they are not sufficient to completely eliminate these effects and implement the flexible wage allocation.

The goal of optimal policy in this environment is to decrease the amplitude of the fluctuations of the equilibrium wage in states of the world where the wage is above the lower bound. By decreasing the equilibrium wage in such a state, we reduce the exposure to the endogenously determined lower bound on wages in the future. To accomplish this goal, we perform a numerical exercise and show that the optimal labor income tax acquires counter-cyclical properties and that the optimal capital income tax acquires pro-cyclical properties.

Furthermore, a predominant feature of models with wage rigidities is the existence of involuntary unemployment. Whenever the wage in a particular state is at the lower bound, households are willing to pay a positive price in order to increase the quantity of labor worked above the quantity firms want to hire. In order words, the multiplier on the rationing constraint is positive. However, we show that optimal policy always eliminates involuntary unemployment. In such a state of the world, a higher labor income tax decreases the quantity of labor households want to supply at each pre-tax wage and, by choosing the correct labor income tax, the quantity of labor households want to supply equals the quantity of labor firms demand at a pre-tax wage equal to the lower bound. Nevertheless, this quantity of labor is below the flexible wage counterpart.

Future work is necessary to fully understand the costs of downward rigid wages. One exercise we will do is to study the welfare costs of following a fiscal policy that would be optimal if wages were flexible, i.e., uniform labor income taxation and zero capital income taxes. Such an exercise will also allow for a deeper understanding of the impact the optimal fiscal policy we propose has on the behavior of macroeconomic variables, particularly, the amplitude of their fluctuations.

References

[1] Adão, B., I. H. Correia, and P. Teles (2009) ”On the relevance of exchange rate regimes for stabilization policy”, *Journal of Economic Theory*, 144, 1468-1488.

[2] Adão, B., I. H. Correia, and P. Teles (2010) ”Wage and Price Rigidity in a Monetary Union”, *Open Economies Review*, 21, 109-126.

[3] Akerlof, G. A., W. Dickens, and G. Perry (1996), ”The Macroeconomics of Low Inflation”, *Brookings Papers on Economic Activity*, 1-59.

[4] Altonji, J. G. and P. J. Devereux (2000), ”The Extent and Consequences of Downward Nominal Wage Rigidity”, *Research and Labor Economics*, 19, 383-431.

[5] Armenter, R. (2008), ”A Note on Incomplete Factor Taxation”, *Journal of Public Economics*, 92, 2275-2281.

[6] Babecky, J., P. Du Caju, T. Kosma, M. Lawless, J. Messina and , T. Rõõm (2009) ”Downward Nominal and Real Wage Rigidity: Survey Evidence from European Firms”, ECB Working Paper 1105

[7] Barro, Robert J. and J. Furman (2018), ”The Macroeconomic Effects of the 2017 Tax Reform”, *Brookings Papers on Economic Activity*, Spring, 257-345.

[8] Belchior, D. and C. Reis (2024), ”Optimal Prudential Taxation”, Working Paper.

[9] Bewley, T. F., ”A Depressed Labor Market as Explained by Participants” (1995), *The American Economic Review*, 85, 250-254.

[10] Bianchi, J. (2011) ”Overborrowing and systemic externalities in the business cycle”, *American Economic Review*, 101(7), 3400-3426.

[11] Boar, C. and V. Midrigan (2022), ”Efficient Redistribution”, *Journal of Monetary Economics*, 131, 78-91.

[12] Campbell, C. M. and K. Kamlani (1997), "The Reasons for Wage Rigidity: Evidence From a Survey of Firms", *The Quarterly Journal of Economics*, 112, 759-789.

[13] Card, D. and D. Hyslop (1997), "Does Inflation Grease the Wheels of the Labor Market?" in *Reducing inflation: Motivation and Strategy*, University of Chicago Press, 71-122.

[14] Chamley, C. (1986) "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives", *Econometrica*, 54, 607-622.

[15] Chari, V. V., L. J. Christiano, and P. J. Kehoe (1991) "Optimal Fiscal and Monetary Policy: Some Recent Results", *Journal of Money, Credit, and Banking*, 23, 519-539.

[16] Chari, V. V., L. J. Christiano, and P. J. Kehoe (1994) "Optimal Fiscal in a Business Cycle Model", *Journal of Political Economy*, 102, 617-652.

[17] Chari, V. V. and P. J. Kehoe (1999) "Optimal Fiscal and Monetary Policy", NBER Working Paper 6891.

[18] Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2007) "Business Cycle Accounting" *Econometrica*, 75, 781-836.

[19] Chari, V. V., J. P. Nicolini, and P. Teles (2020) "Optimal Capital Taxation Revisited", *Journal of Monetary Economics*, 116, 147-165.

[20] Correia, I. H. (1996) "Should Capital Income be Taxed in the Steady State?", *Journal of Public Economics*, 60, 147-151.

[21] Correia, I. H., E. Farhi, J. P. Nicolini, and P. Teles (2013) "Unconvential Fiscal Policy at the Zero Bound", *American Economic Review*, 103(4), 1172-1211.

[22] Dickens, W. T., L. Goette, E. L. Groshen, S. Holden, J. Messina, M.E. Schweitzer, J. Turunen, and M. E. Ward (2007), "How wages change: micro evidence from the International Wage Flexibility Project", *Journal of Economic Perspectives*, 21, 195-214.

[23] Elsby, M. W. L. and G. Solon (2019) "How Prevalent is Downward Rigidity in Nominal

Wages? International Evidence from Payroll Records and Pay Slips”, *Journal of Economic Perspectives*, 33(3), 185-201.

[24] Farhi, E., G. Gopinath, and O. Itskhoki (2014) ”Fiscal devaluations”, *The Review of Economic Studies*, 81(2), 725-760.

[25] Gottschalk, P. (2005), ”Downward Nominal Wage Flexibility: Real or Measurement Error?”, *The Review of Economics and Statistics*, 87, 556-568.

[26] Hazell, J. and B. Taska (2024) ”Downward Rigidity in the Wage for New Hires”, *American Economic Review (revise and resubmit)*.

[27] Holden, S. and F. Wulfsberg (2008), ”Downward Nominal Wage Rigidity in the OECD”, *The B.E. Journal of Macroeconomics*, 1: Article 15.

[28] Holden, S. and F. Wulfsberg (2009), ”How Strong is the Macroeconomic Case for Downward Real Wage Rigidity?”, *Journal of Monetary Economics*, 56, 605-615.

[29] Judd, K. L. (1985) ”Redistributive Taxation in a Simple Perfect Foresight Model”, *Journal of Public Economics*, 28, 59-83.

[30] Karabarbounis, L. (2013) ”The Labor Wedge: MRS vs. MPL”, NBER Working Paper 19015.

[31] Lebow, D. E., R. E. Saks, and B. A. Wilson (2003), ”Downward Nominal Wage Rigidity: Evidence from the Employment Cost Index”, *Advances in Macroeconomics*, 3(1): Article 2.

[32] Lucas, R. E., N. Stokey (1983) ”Optimal Fiscal and Monetary Policy in a World Without Capital”, *Journal of Monetary Economics*, 12, 55-93.

[33] Schmitt-Grohé, S. and M. Uribe (2016) ”Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment,” *Journal of Political Economy*, 5, 1466-1514..

[34] Shimer, R. (2009) ”Convergence in Macroeconomics: The Labor Wedge”, *American Economic Journal: Macroeconomics*, 1, 280-297.

[35] Wolf, M. (2020) ”Pecuniary Externalities in Economics with Downward Wage Rigidity”, *Journal of Monetary Economics*, 116, 219-235.

[36] Zhu, X. (1992) ”Optimal Fiscal Policy in a Stochastic Growth Model”, *Journal of Economic Theory*, 58, 250-289.

Appendix

A Proof of Proposition 1

Our proof of Proposition (1) starts with the two Lemmas that follow.

Lemma 1. An allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$, Lagrange multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ can be attained as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$ if and only if they satisfy the resource constraints (1) for all (t, s^t), the constraints on the marginal product of labor (18), the implementability condition

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[u'(c(s^t))c(s^t) - \theta h'(n(s^t))n(s^t)] = \mathcal{W}_0 + \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[\lambda(s^t)n(s^t) + u'(c(s^t))T(s^t)], \tag{23}$$

and the complementary slackness conditions

$$\begin{aligned}
&\lambda(s^t)[A(s^t)F_n(k(s^{t-1}), n(s^t)) - \gamma A(s^{t-1})F_n(k(s^{t-2}), n(s^{t-1}))] = 0, \quad \forall(t \geq 1, s^t)\\
&\lambda(s^0)[A(s^0)F_n(k(s^{-1}), n(s^0)) - \gamma w(s^{-1})] = 0 \quad \text{for} \quad t = 0,\\
&\lambda(s^t) \geq 0, \quad \forall(t, s^t).
\end{aligned} \tag{24}$$

The implementability condition (23) suggests that lump-sum transfers $T(s^t)$ and Lagrange multipliers $\lambda(s^t)$ play a similar role in the characterization of attainable allocations. To see this, consider an equilibrium with downward rigid wages where $\lambda(s^t) > 0$ for some history s^t. This implies that the labor income tax rate at history s^t is such that

$$\frac{\theta h'(n^t)}{u'(c(s^t))} < (1 - \tau_n(s^t))w(s^t).$$

An alternative implementation of the same allocation consists of a higher labor income tax rate $\tau_n(s^t)$ such that the marginal rate of substitution between labor and consumption equals the after-tax wage and a higher lump-sum transfer. This alternative allocation has a Lagrange multiplier equal to zero. Therefore, households are effectively receiving a lump-sum transfer when $\lambda(s^t) > 0$. This intuition leads to the following Lemma[17].

Lemma 2. Let the allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$, multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ be attainable as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$. There exists a sequence of lump-sum transfers $\{\{\hat{T}(s^t)\}_{s^t}\}_{t\geq 0}$ with $\hat{T}(s^t) \geq T(s^t)$ for all (t, s^t) such that the allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$ and lump-sum transfers $\{\{\hat{T}(s^t)\}_{s^t}\}_{t\geq 0}$ are also attainable as part of an equilibrium with downward rigid wages with the initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$ where the Lagrange multipliers are $\hat{\lambda}(s^t) = 0$ for all (t, s^t).

Proof of Lemma 1. In one direction, take an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$. Conditions (3) - (16) are satisfied and we must show that (1), (18), (23), and (24). Use the input market clearing conditions (14) and (15) to substitute away the firm allocation from the equilibrium conditions. Doing this in the output market clearing conditions (13) yields (1). Doing this in the firms' first-order condition (10) yields the wage $w(s^t)$ in terms of $(k(s^{t-1}), n(s^t))$. Next, substitute away the resulting equation in the downward wage rigidity constraints (3) and the complementary slackness conditions (16). This yields (18) and (24), respectively. The households' consolidated budget constraint is

$$\sum_{t=0}^{\infty}\sum_{s^t}\left(\prod_{j=0}^{t} q(s_j|s^{j-1})\right)[c(s^t) - (1-\tau_n(s^t))w(s^t)n(s^t)] =$$
$$(1-\delta + (1-\tau_k(s^{-1}))r(s^0))k(s^{-1}) + b(s^{-1}) + \sum_{t=0}^{\infty}\sum_{s^t}\left(\prod_{j=0}^{t} q(s_j|s^{j-1})\right)T(s^t),$$

[17]Our environment allows for lump-sum transfers because we want to make allocations attainable with $\lambda(s^t) = 0$ for all (t, s^t). Without lump-sum transfers, this would only be necessarily true for the allocation that solves the optimal taxation problem

where $q(s_0|s^{-1}) = 1$. Use the Euler equation (7) to substitute away the prices of state-contingent assets, the intra-temporal marginal conditions (8) to substitute away after-tax wages, and the firms' first-order conditions (11) to substitute away the capital rent at $t = 0$. This yields (23).

In the other direction, take an allocation allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$, multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ that satisfy conditions (1), (18), (23), and (24). We need to find a firm allocation, state-contingent assets, prices, and policies, such that conditions (3) - (16) are satisfied. The input market clearing conditions (14) and (15) pin down the firm allocation $\{\{k^d(s^t), n(s^t)\}_{s^t}\}_{t\geq 0}$. The firms' first-order conditions (10) and (11) pin down pre-tax wages $\{\{w(s^t)\}_{s^t}\}_{t\geq 0}$ and capital rents $\{\{r(s^t)\}_{s^t}\}_{t\geq 0}$, respectively. The Euler equations (6) and (7) pin down ex-ante capital income tax rates $\{\{\tau_k(s^t)\}_{s^t}\}_{t\geq 0}$ and state-contingent asset prices $\{\{q(s_{t+1}|s^t)\}_{s^t}\}_{t\geq 0}$, respectively. The intra-temporal marginal conditions (8) pin down labor income tax rates $\{\{\tau_n(s^t)\}_{s^t}\}_{t\geq 0}$. The households' budget constraints (4) pin down state-contingent assets $\{\{b(s_{t+1}|s^t)\}_{s^t}\}_{t\geq 0}$. The complementary slackness conditions that characterize the solution to the utility-maximization problem (9) are satisfied since (15) holds. The output market clearing conditions (13) are satisfied since (1) holds, and since (14) and (15) hold. The downward wage rigidity constraints (3) complementary slackness conditions (16) are satisfied since (18) and (24) hold. Finally, the government's budget constraints (12) are satisfied by Walras' law. ■

Proof of Lemma 2. Take an allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$, multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ that can be attained as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$. It follows from Lemma (1) that conditions (1), (18), (23), and (24) are satisfied. Now, define the new sequence of lump-sum transfers, $\{\{\hat{T}(s^t)\}_{s^t}\}_{t\geq 0}$ as follows:

$$\hat{T}(s^0) = T(s^0) + \frac{1}{u'(c(s^0))} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \lambda(s^t) n(s^t)$$

$$\hat{T}(s^t) = T(s^t), \quad \forall (t \geq 1, s^t).$$

It suffices to show that the allocation, the new sequence of lump-sum transfers, and the sequence of multipliers $\{\{\hat{\lambda}(s^t)\}_{s^t}\}_{t\geq 0}$, where $\hat{\lambda}(s^t) = 0$ for all (t, s^t) satisfy conditions (1), (18), (23), and (24). The resource constraints (1) and the constraints on the marginal product of labor (18) are satisfied, since the allocation is attainable as an equilibrium with downward rigid wages. Conditions (24) are satisfied with $\hat{\lambda}(s^t) = 0$ for all (t, s^t). To see that the implementability condition (23) is satisfied, observe that

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[\hat{\lambda}(s^t)n(s^t) + u'(c(s^t))\hat{T}(s^t)] = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)u'(c(s^t))\hat{T}(s^t)$$
$$= u'(c(s^0))[T(s^0) + \frac{1}{u'(c(s^0))}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)\lambda(s^t)n(s^t)] +$$
$$\sum_{t=1}^{\infty}\sum_{s^t}\beta^t\pi(s^t)u'(c(s^t))T(s^t)$$
$$= \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[\lambda(s^t)n(s^t) + u'(c(s^t))T(s^t)].$$

This completes the proof. ■

Lemma (1) and Lemma (2) yield the following corollary.

Corollary. An allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$ and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ can be attained as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^-$ if and only if they satisfy the resource constraints (1) for all (t, s^t), constraints on the marginal product of labor (18) and the implementability condition

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[u'(c(s^t))c(s^t) - \theta h'(n(s^t))n(s^t)] = \mathcal{W}_0 + \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)u'(c(s^t))T(s^t). \tag{25}$$

To complete the proof of Proposition (1), we need only to impose the non-negativity of lump-sum transfers.

B Proof of Proposition 2

Let $\mu > 0$ be the Lagrange multiplier on (17) and $\beta^t \pi(s^t)\eta(s^t) \geq 0$ be the present value Lagrange multiplier on (18) at history s^t. The Ramsey allocation must satisfy the necessary first-order conditions

$$\begin{aligned}\frac{\theta h'(n(s^t))}{u'(c(s^t))} =& \frac{1+\mu(1-\sigma)}{1+\mu(1+\psi)} A(s^t) F_n(k(s^{t-1}), n(s^t)) + \frac{A(s^t)F_{nn}(k(s^{t-1}), n(s^t))}{u'(c(s^t))[1+\mu(1+\psi)]}\eta(s^t) - \\ & \beta\gamma \frac{A(s^t)F_{nn}(k(s^{t-1}), n(s^t))}{u'(c(s^t))[1+\mu(1+\psi)]} \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t)\eta(s^{t+1})\end{aligned}$$

and

$$\begin{aligned}u'(c(s^t)) =& \beta \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) u'(c(s^{t+1}))[1-\delta + A(s^{t+1})F_k(k(s^t), n(s^{t+1}))] + \\ & \beta \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) \frac{A(s^{t+1})F_{nk}(k(s^t), n(s^{t+1}))}{1+\mu(1-\sigma)} \eta(s^{t+1}) - \\ & \beta^2\gamma \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) \frac{A(s^{t+1})F_{nk}(k(s^t), n(s^{t+1}))}{1+\mu(1-\sigma)} \sum_{s_{t+2}|s^{t+1}} \pi(s_{t+2}|s^{t+1})\eta(s^{t+2})\end{aligned}$$

for all (t, s^t). An equilibrium allocation with downward rigid wages must satisfy, for all (t, s^t),

$$\frac{\theta h'(n(s^t))}{u'(c^t))} = (1-\tau_n(s^t))A(s^t)F_n(k(s^{t-1}), n(s^t)) - \frac{\lambda(s^t)}{u'(c(s^t))}$$

and

$$u'(c(s^t)) = \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t) u'(c(s^{t+1}))[1-\delta + (1-\tau_k(s^t))A(s^{t+1})F_k(k(s^t), n(s^{t+1}))].$$

Lemma (2) shows that the Ramsey allocation can be implemented with $\lambda(s^t) = 0$ for all s^t. Therefore, the state-contingent labor income tax at history s^t that implements the Ramsey allocation is implicitly defined by

$$1-\tau_n(s^t) = \frac{1+\mu(1-\sigma)}{1+\mu(1+\psi)} + \frac{F_{nn}(k(s^{t-1}),n(s^t))}{F_n(k(s^{t-1}),n(s^t))}\frac{\eta(s^t)}{u'(c(s^t))[1+\mu(1+\psi)]} - \frac{\beta\gamma}{u'(c(s^t))[1+\mu(1+\psi)]}\frac{F_{nn}(k(s^{t-1}),n(s^t))}{F_n(k(s^{t-1}),n(s^t))}\sum_{s_{t+1}|s^t}\pi(s_{t+1}|s^t)\eta(s^{t+1}).$$

Solving for $\tau_n(s^t)$ yields (19). And the ex-ante capital income tax at history s^t that implements the Ramsey allocation is implicitly defined by

$$\tau_k(s^t))\beta\sum_{s_{t+1}\in S}\pi(s_{t+1}|s^t)u'(c(s^{t+1}))A(s^{t+1})F_k(k(s^t),n(s^{t+1})) = \beta\sum_{s_{t+1}|s^t}\pi(s_{t+1}|s^t)\frac{A(s^{t+1})F_{nk}(k(s^t),n(s^{t+1}))}{1+\mu(1-\sigma)}\eta(s^{t+1}) - \beta^2\gamma\sum_{s_{t+1}|s^t}\pi(s_{t+1}|s^t)\frac{A(s^{t+1})F_{nk}(k(s^t),n(s^{t+1}))}{1+\mu(1-\sigma)}\sum_{s_{t+2}|s^{t+1}}\pi(s_{t+2}|s^{t+1})\eta(s^{t+2}).$$

Solving for $\tau_k(s^t)$ yields (20).

This completes the proof of Proposition (2).