Income Taxation over the Business Cycle with Wage Rigidities[*]

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This draft: June 1, 2024

First draft: April 24, 2024

Abstract

We study how optimal income taxes behave over the business cycle in the presence of downward rigid wages. We determine the optimal Ramsey taxes within the context of the standard neoclassical general equilibrium model and find that the optimal labor income tax exhibits two properties: (i) it should increase when the wages are at the lower bound because it increases tax revenue without introducing additional distortions; (ii) the optimal labor income tax incorporates the fact that the lower bound for the next period depends on the current wage. Thus, a lower labor income tax can reduce the exposure to the lower bound in the future. The optimal capital income tax exhibits similar properties, but the effects are in the reverse direction since the capital income tax is set one period in advance, thus influencing the stock of capital of the period in which it is collected, and also because labor and capital and complements in production. Finally, we solve a numerical exercise to illustrate these properties and conclude that the optimal labor income tax behaves counter-cyclically to output, and that the optimal capital income tax behaves pro-cyclically.

Keywords: Optimal fiscal policy, Downward wage rigidity, Counter-cyclical labor income tax
JEL Codes: H21, E24, E32, J64

[*]Daniel Belchior is thankful to his advisor V.V. Chari, as well as Larry Jones and Christopher Phelan for all their support, encouragement, and very insightful discussions. Belchior also thanks Ricardo Alves Monteiro, Amol, Xavier Rainero, and Mateus Santos for all the helpful discussions. All errors that remain are our own.

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1 Introduction

The classical result in the literature of optimal taxation is the Chamley-Judd result- after Judd (1985) and Chamley (1986). This result states that capital income should not be taxed in the long-run. Over the business cycle, optimal fiscal policy is characterized by a similar behavior. Chari, Christiano, and Kehoe (1991) show that optimal fiscal policy over the business cycle exhibits two properties: (i) the optimal labor income tax is roughly constant; and (ii) the optimal capital income is approximately equal to zero on average.[1] It is also widely known that if preferences are standard preferences in macroeconomics[2], then the optimal labor income tax is exactly constant over the business cycle and that the optimal capital income tax is exactly equal to zero.

In this paper, we augment the standard framework where these results hold and study how labor market frictions affect optimal fiscal policy. Particularly, we are interested in downward rigid wages as in Schmitt-Grohé and Uribe (2016). This friction is characterized by the fact that the wage in every period cannot fall below an endogenously determined lower bound, which corresponds to a fraction of the wage in the previous period. Since agents are atomistic, they fail to recognize that their current decisions affect the lower bound on wages in the next period. Therefore, there is a pecuniary externality. Furthermore, the existence of a price floor in the labor market makes it possible to have involuntary unemployment along the equilibrium path for a given arbitrary fiscal policy.

To address these problems, optimal fiscal policy cannot be used as under flexible prices. Instead, it must change along the business cycle depending on whether the equilibrium wage is at the lower bound or not, even if preferences are standard preferences in macroe-

[1] For more references on these results, see Chari, Christiano, and Kehoe (1994), Chari and Kehoe (1999), and Chari, Nicolini, and Teles (2020).

[2] Standard preferences in macroeconomics are characterized by the following three properties: (i) preferences are separable in consumption and leisure; (ii), the inter-temporal elasticity of substitution, $-\left(u''(c)/u'(c)\right)c$, is constant; and (iii) the Frisch elasticity of labor supply is constant.

conomics. We show that the optimal labor income tax is used to lower the current wage in order to reduce the exposure to the endogenous lower bound in the future. However, if the current wage is at the lower bound and there is involuntary unemployment, then the optimal labor income should in such a way that the pre-tax wage (the lower bound) clears the labor market. This is optimal because increasing the labor income tax in this situation does not change the labor market outcome, which means that it is possible to raise revenue without introducing additional distortions. Therefore, we show that the labor income tax should be low when the current wage is above its lower bound and it should be high when the current wage is at the lower bound. In order words, the optimal labor income tax behaves counter-cyclically. Moreover, the capital income tax also plays a role because the marginal product of capital depends positively on the stock of capital. If the current wage is at the lower bound, then the capital income tax collected in the current period is low. Since this event was assigned a positive probability in the previous period, the ex-ante capital income tax should be used to affect investment and, as a result, the stock of capital. A low capital income tax induces households to invest more in capital, leading to a higher marginal product of capital in the current perioda and offsetting the negative effects of the lower bound on wages. But if in the current period, there is a positive probability that the wage is going to be at the lower bound in the next period, then the ex-ante capital income tax collected in the current period should be high. This would have reduced investment in the previous period and contributed to a lower marginal product of labor in the current period - since lower investment results in a lower capital stock, - thus reducing the exposure to the endogenous lower bound on wages in the future. To summarize, we show that the optimal ex-ante capital income tax behaves pro-cyclically.

We derive these results in a standard framework in the literature of optimal fiscal policy. We consider a real dynamic stochastic general equilibrium model with endogenous labor, capital accumulation, complete markets, and a government that only has access to a state-contingent labor income taxes and an ex-ante capital income tax. However, we introduce a friction in the labor market in the form of downward rigid wages that we borrow from Schmitt-Grohé and Uribe (2016). In this environment, downward rigid wages effectively work as an endogenous price floor in the labor market, which is occasionally binding. When the wage hits the lower bound, the quantity of labor that solves the firms' profit-maximization problem is less than the quantity of labor that solves the (unconstrained) utility-maximization problem[3] - there is involuntary unemployment. Moreover, the quantity of labor worked is determined by the firms' demand. We study the general equilibrium effects of downward rigid wages by introducing an additional constraint on the households' consumption set. This constraint states that households cannot choose a quantity of labor above some exogenously given level, which, in equilibrium, represents the quantity of labor that solves the firms' profit-maximization problem. Therefore, the interpretation of the additional constraint is that households cannot choose to work more than the quantity of labor that firms wish to use in production. This constraint is the novelty in our environment. Without this constraint, the marginal rate of substitution between labor and consumption is strictly less than the after-tax wage when the wage hits the lower bound. Consequently, the solution concept must include a complementary slackness condition expressing this relation between the downward wage rigidity constraint and the difference between the marginal rate of substitution between labor and consumption and the after-tax wage. This complementary slackness condition appears naturally as an equilibrium condition once we introduce the additional constraint on the consumption set. Additionally, the introduction of this constraint ensures that the quan-

[3]We say *unconstrained*, because we will introduce a rationing constraint in the consumption set. See the discussion below.

tity of labor that solves the households' utility maximization problem is always equal to the quantity of labor that solves the firms' profit-maximization problem. Nevertheless, involuntary unemployment still exists in our environment, although it cannot be directly measured[4]. When this constraint is binding, the Lagrange multiplier is strictly positive and it represents the shadow price households are willing to pay to be able to work more at the market wage rate.

The empirical relevance of downward rigid wages is well established in the literature. At a micro-level, Babecky, Du Caju, Kosma, Lawless, Messina, and Rõõm (2009) use a survey of European firms to determine whether downward nominal and real wage rigidities are relevant. They conclude that the workforce composition of firms as well as the institutional framework of the labor market can influence the strength of downward wage rigidities. For example, the authors find that there is a positive correlation between the existence of union coverage and the prevalence of downward rigid real wages. Hazell and Taska (2023) focus on the wage of new hires and show that they are downward rigid, but upward flexible. At a macro-level, Schmitt-Grohé and Uribe show that wages are downward rigid in the peripheral European countries and Argentina[5].

This paper relates to the literature of fiscal devaluations. Adão, Correia, and Teles (2008 and 2009) show that fiscal policy can be used to implement equilibrium allocations with flexible price under sticky prices. They highlight the role of a payroll tax to ensure that a flexible wage allocation can be implemented with constant wages. In a similar exercise, Farhi, Gopinath, and Itskhoki (2014) show that fiscal policy can be used to replicate the allocations that are obtainable with nominal devaluations. They also emphasize the role

[4]We could measure involuntary unemployment by solving the household's utility-maximization problem without the rationing constraint and compute the difference between the choices of labor that solve the problem with and without the rationing constraint.

[5]See references in Schmitt-Grohé and Uribe (2016) to find more evidence on downward wage rigidity in developed countries, such as the U.S.

of the payroll tax to overcome the friction imposed by wage stickiness. Our environment is a closed economy environment, but these results are still true. If we enlarge our set of fiscal instruments to include payroll taxes, the distortion imposed by downward rigid wages ceases to be relevant in the sense that it is possible to implement the flexible wage allocation. In this setting, optimal fiscal policy would be characterized by a decrease in the payroll tax rate and an increase in the labor income tax rate in histories where the wage cannot fully adjust downward. By abstracting from payroll taxes, we cannot implement the flexible wage allocation. Optimal fiscal policy without payroll taxes is characterized by an increase in the labor income tax rate in histories where the wage cannot fully adjust downward until the gap in the labor market is closed. The interpretation is that in such histories, the labor market outcome is uniquely determined by firms' demand. As a result, the government can collect higher revenue without raising additional distortions.

This paper also relates to the literature of the labor wedge, the gap between the marginal rate of substitution between labor and consumption and the marginal product of labor. Shimer (2009) measures the labor wedge in the United States and shows that it is counter-cyclical. Karabarbounis (2013) decomposes the labor wedge into the firm component of the labor wedge and the household component and shows that most of the variation of the labor wedge is due to the household component. In our environment, the optimal labor wedge exhibits a counter-cyclical behavior. This is not surprising since the present of downward rigid wages makes it so that whenever the current wage is at the lower bound, the marginal rate of substitution between labor and consumption is strictly less than the after-tax real wage (unless the optimal policy is implemented). Therefore, the very nature of the friction imposed generates a counter-cyclical gap between the marginal rate of substitution between labor and consumption and the marginal product of labor. Since the role of the optimal labor income tax is to ensure that the marginal rate of substitution between labor and consumption is always equal to the after-tax real wage, it follows that the optimal labor wedge in our environment is counter-cyclical.

The paper is organized as follows. Section 2 introduces the environment and addresses the implications of downward rigid wages by introducing a rationing constraint on the households' consumption set. Section 3 includes the optimal taxation problem and the theoretical results about optimal income taxation policy with downward rigid wages. In section 4, we perform a numerical exercise and explore the cyclical properties of the optimal income taxes. Section 5 concludes. All proofs are relegated to the Appendix.

2 Environment

Consider an infinitely-lived representative agent closed economy. Every period $t \geq 0$ a random variable s_t is drawn from the finite set $S = \{0, 1, ..., S\}$. The exogenous state of the economy at t is $s^t = (s_0, s_1, ..., s_t)$, which represents the history of realizations up until period t. Also, the probability of reaching state s^t is $\pi(s^t)$. We assume that the initial state s_0 is given. The exogenous state s^t determines aggregate productivity and government spending. Let $A(s^t)$ and $g(s^t)$ denote aggregate productivity and government spending, respectively, if the history at t is s^t.

Every (t, s^t), a continuum of measure one of identical firms transforms capital, $k(s^{t-1})$, and labor, $n(s^t)$, into a single final good using a technology represented by

$$A(s^t)F(k(s^{t-1}), n(s^t)),$$

where $F(\cdot)$ is a production function strictly increasing, strictly concave, twice continuously differentiable, and satisfying the Inada conditions. The final good is used for private consumption, $c(s^t)$, government spending, $g(s^t)$, and investment, $k(s^t) - (1-\delta)k(s^{t-1})$, according to the resource constraint

$$c(s^t) + g(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) = A(s^t)F(k(s^{t-1}), n(s^t)). \tag{1}$$

There is a continuum of measure one of identical households with preferences over streams of state contingent consumption and labor, $\{\{c(s^t), n(s^t)\}_{s^t}\}_{t\geq 0}$, represented by the lifetime utility function

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t \pi(s^t)[u(c(s^t)) - \theta h(n(s^t))], \tag{2}$$

where $\beta \in (0,1)$ is the subjective discount factor and $\theta > 0$ is the disutility of labor parameter. The period utility function $u(\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable, and satisfies the Inada conditions. Similarly, $h(\cdot)$ is strictly increasing, strictly convex, twice continuously differentiable, and satisfies the Inada conditions. Furthermore, we assume that preferences are standard in macroeconomics.

Assumption. The period utility function has a constant coefficient of relative risk aversion in consumption and a constant Frisch elasticity of labor supply. That is[6],

$$-\frac{u''(c(s^t))}{u'(c(s^t))}c(s^t) = \sigma \quad \text{and} \quad \frac{h''(n(s^t))}{h'(n(s^t))}n(s^t) = \psi \quad \forall (t, s^t).$$

There is also a government that must finance a stream of exogenously given government spending, $\{\{g(s^t)\}_{s^t}\}_{t\geq 0}$, and initial debt, $b(s^0)$. We assume that the government has access to standard income taxes - a state-contingent labor income tax, $\tau_n(s^t)$, and an ex-

[6]The Frisch elasticity of labor supply is $1/\psi$.

ante capital income tax, $\tau_k(s^{t-1})$. We further assume that the government can issue state-contingent claims, $b(s_{t+1}|s^t)$. Lastly, we assume that the government can make state-contingent transfers, $T(s^t)$, to households, but it cannot collect lump-sum taxes.

Finally, the environment is characterized by the existence of a friction in the labor market. Specifically, we impose that pre-tax wages are downward rigid. That is,

$$w(s^t) \geq \gamma w(s^{t-1}), \qquad \forall \quad (t, s^t), \tag{3}$$

where $\gamma \in [0, 1)$ measures the degree of downward wage rigidity. In our environment, downward wage rigidity represents a pecuniary externality because the labor market outcome at $(t-1, s^{t-1})$ affects the labor market outcomes that can be attained at all s^t. Yet, agents fail to incorporate these effects in their decision problems. This can cause involuntary unemployment at some s^t if $w(s^{t-1})$ is high enough and, as a result, $w(s^t)$ cannot fully adjust downwards in response to a low realization of the aggregate productivity shock. We assume that the initial lower bound, $\gamma w(s^{-1})$, is exogenously given.

2.1 The households' problem

Every (t, s^t), households receive an after-tax wage $(1 - \tau_n(s^t))w(s^t)$ for every unit of labor $n(s^t)$, where $\tau_n(s^t)$ is the labor income tax rate, and also receive an after-tax capital rent $(1 - \tau_k(s^{t-1}))r(s^t)$ for every unit of capital $k(s^{t-1})$ rented out to firms, where $\tau_k(s^{t-1})$ is the ex-ante capital income tax rate. Furthermore, households receive a payment of one unit of output from each Arrow-Debreu security purchased in the previous period that pay only if s^t is realized. Finally, they receive lump-sum transfers, $T(s^t)$, from the government. Households use income to finance consumption expenditures, $c(s^t)$, capital investment expenditures, $k(s^t) - (1-\delta)k(s^{t-1})$, and purchases of state-contingent claims, $b(s_{t+1}|s^t)$, that cost $q(s_{t+1}|s^t)$ units of output in (t, s^t) and pay one unit of output in $t+1$ if s_{t+1} is realized and zero otherwise. The budget constraints are

$$\begin{aligned} c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) + \sum_{s_{t+1}\in S} q(s_{t+1}|s^t)b(s_{t+1}|s^t) = \\ (1-\tau_n(s^t))w(s^t)n(s^t) + (1-\tau_k(s^{t-1}))r(s^t)k(s^{t-1}) + b(s_t|s^{t-1}) + T(s^t), \quad \forall(t, s^t), \end{aligned} \tag{4}$$

together with a no-Ponzi games condition.

In our environment, however, we must introduce an additional restriction to the households' consumption set, which is

$$n(s^t) \leq n^d(s^t), \quad \forall(t, s^t). \tag{5}$$

The interpretation of this constraint is that, at any (t, s^t), the quantity of labor that households choose to work cannot exceed some value $n^d(s^t)$. Households take $n^d(s^t)$ as given, but this is an object that is determined by our solution concept and it represents the quantity of labor that firms demand at (t, s^t)[7]. The role of this constraint is to account for the fact that in states where wages cannot fully adjust downwards and (3) is binding, the quantity of labor is uniquely determined by firms. Households are off the labor supply schedule and there is involuntary unemployment. This issue is typically addressed by imposing a complementary slackness condition that states that when the the wage cannot fully adjust downwards, the quantity of labor worked is determined by firms[8]. Our additional constraint makes the complementary slackness condition a necessary condition for the solution to the utility-maximization problem. With this constraint we ensure

[7]We can argue that this constraint is also part of the households' consumption set even when wages are flexible. The difference, relative to our environment, is that when wages are flexible, this constraint is never binding.

[8]See, for example, Schmitt-Grohé and Uribe (2016).

that the quantity of labor that solves the utility-maximization problem always equals the quantity of labor that solves the profit-maximization problem. But this does not mean that there is no involuntary unemployment. In this setting, there is involuntary unemployment when (5) is binding, with the Lagrange multiplier on this constraint representing the price households are willing to pay in order to be able to work one additional marginal unit of labor.

The utility-maximization problem consists in choosing the state-contingent sequence of consumption, labor, capital and Arrow-Debreu securities that maximizes (2) subject to (4), (5), and a no-Ponzi games condition, taking the sequences of prices, taxes, lump-sum transfers, and upper bounds on labor, as well as initial conditions $k(s^{-1})$ and $b(s^{-1})$. Let $\beta^t \pi(s^t)\lambda(s^t)$ be the present-value multiplier on (5). The solution to the utility-maximization problem must satisfy the Euler equations

$$u'(c(s^t)) = \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) u'(c(s^{t+1}))[1 - \delta + (1 - \tau_k(s^t)) r(s^{t+1})], \quad \forall (t, s^t) \tag{6}$$

$$u'(c(s^t)) = \beta \frac{\pi(s_{t+1}|s^t)}{q(s_{t+1}|s^t)} u'(c(s^{t+1})), \quad \forall (t, s^t), \tag{7}$$

and the intra-temporal marginal conditions and complementary slackness conditions

$$\frac{\theta h'(n(s^t))}{u'(c(s^t))} = (1 - \tau_n(s^t)) w(s^t) - \frac{\lambda(s^t)}{u'(c(s^t))} \quad \forall (t, s^t) \tag{8}$$

$$\lambda(s^t)[n(s^t) - n^d(s^t)] = 0 \quad \text{and} \quad \lambda(s^t) \geq 0, \quad \forall (t, s^t). \tag{9}$$

Equations (8) and (9) are the novelty of our environment, since they result from the additional constraint on the consumption set (5). They allow for the consideration of involuntary unemployment while ensuring that the quantity of labor that solves the utility-maximization problem does not necessarily belong to the labor supply schedule. To see this, consider that there is some history s^t such that $n^d(s^t)$ is low enough. As a result, $\lambda(s^t) \geq 0$ and

$$\theta h'(n(s^t)) < u'(c(s^t))(1-\tau_n(s^t))w(s^t).$$

This shows that, at the given after-tax wage, the households' disutility of one additional marginal unit of labor is less than the utility brought about $(1-\tau_n(s^t))w(s^t)$ additional units of consumption. Consequently, households wish to increase the quantity of labor, but they cannot. Whenever (5) is binding, the solution to the utility-maximization problem is off the labor supply schedule and there is involuntary unemployment.

2.2 The firms' problem

Firms solve a sequence of static problems. Every (t, s^t), they hire labor, $n^d(s^t)$, at the given pre-tax wage $w(s^t)$, and rent capital, $k^d(s^t)$, at the given pre-tax rent $r(s^t)$ to produce a final good with the highest possible profit. Profit is

$$A(s^t)F(k^d(s^t), n^d(s^t)) - w(s^t)n^d(s^t) - r(s^t)k^d(s^t), \quad \forall (t, s^t).$$

The solution to the profit-maximization problem satisfies the standard first-order conditions that state the equality between marginal product of an input and its real remuneration,

$$A(s^t)F_n(k^d(s^t), n^d(s^t)) = w(s^t), \quad \forall(t, s^t) \tag{10}$$

$$A(s^t)F_k(k^d(s^t), n^d(s^t)) = r(s^t), \quad \forall(t, s^t). \tag{11}$$

2.3 The government's budget constraint

Every (t, s^t), the government must finance exogenously given government spending, $g(s^t)$, debt obligations $b(s_t|s^{t-1})$, and lump-sum transfers, $T(s^t)$. To meet this obligations, the government collects state-contingent taxes on labor income and ex-ante taxes on capital income. For every unit of labor rented out to firms, the government collects a tax equal to $tau_n(s^t)w(s^t)$ and for every unit of capital rented out to firms, it collects a tax equal to $\tau_k(s^{t-1})k(s^{t-1})$. Additionally, the government can issue new state-contingent claims $b(s_{t+1}|s^t)$ that are a sold at a price $q(s_{t+1}|s^t)$ and return a payment of one unit of output in $t+1$ if s_{t+1} is realized and zero otherwise. The government's budget constraints are

$$\begin{aligned} g(s^t) + b(s_t|s^{t-1}) + T(s^t) = \tau_n(s^t)w(s^t)n(s^t) + &\tau_k(s^{t-1})r(s^t)k(s^{t-1}) + \\ &\sum_{s_{t+1}\in S} q(s_{t+1}|s^t)b(s_{t+1}|s^t), \quad \forall(t, s^t), \end{aligned} \tag{12}$$

together with a no-Ponzi games condition.

2.4 Market clearing and downward rigid wages

The market clearing conditions in the output and capital markets are standard in our environment. Respectively,

$$c(s^t) + g(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) = A(s^t)F(k^d(s^t), n^d(s^t)), \quad \forall (t, s^t) \tag{13}$$

$$k^d(s^t) = k(s^{t-1}), \quad \forall (t, s^t). \tag{14}$$

The addition of constraint (5) to the households' consumption set implies that the labor market also clears. At every (t, s^t), the wage $w(s^t)$ is such that the quantity of labor that solves the utility-maximization problem equals the quantity of labor that solves the profit-maximization problem. That is,

$$n^d(s^t) = n(s^t), \quad \forall (t, s^t). \tag{15}$$

The downward wage rigidity constraint (3), when binding, plays the role of affecting the quantity of labor traded between households and firms. To understand this role, consider some history s^t where (3) is binding. Since the wage $w(s^t) = \gamma w(s^{t-1})$ cannot adjust downwards, the quantity of labor that satisfies the firms' first-order condition (10) - which defines the upper bound on the quantity of labor households can choose - implies that (5) is binding at the solution to the utility-maximization problem at history s^t. This, in turn, means that $\lambda(s^t) > 0$. A similar argument suggests that if (3) is not binding at some history $\tilde{s}^t$, then the solution to the utility-maximization problem has $\lambda(\tilde{s}^t) = 0$. We summarize this idea with the following complementary slackness condition as a requirement to our solution concept:

$$\lambda(s^t)[w(s^t) - \gamma w(s^{t-1})] = 0, \quad \text{and} \quad \lambda(s^t) \geq 0, \quad \forall (t, s^t). \tag{16}$$

2.5 Solution concept with downward rigid wages

We finish this section with a definition of our solution concept in the presence of downward rigid wages. We refer to this solution concept as an *equilibrium with downward rigid wages*.

Definition 1 (Equilibrium with downward rigid wages)**.** An *equilibrium with downward rigid wages* is an allocation for households $\{\{c(s^t), n(s^t), k(s^t), b(s_{t+1}|s^t)\}_{s^t}\}_{t\geq 0}$, an allocation for firms $\{\{k^d(s^t), n^d(s^t)\}_{s^t}\}_{t\geq 0}$, prices $\{\{w(s^t), r(s^t), q(s_{t+1}|s^t)\}_{s^t}\}_{t\geq 0}$, Lagrange multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and policies $\{\{\tau_n(s^t), \tau_k(s^t), T(s^t)\}_{s^t}\}_{t\geq 0}$, such that, given the sequences $\{\{A(s^t, g(s^t)\}_{s^t}\}_{t\geq 0}$ and initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$, (i) the household allocation solves the utility-maximization problem, (ii) the firm allocation solves the profit maximization problem, (iii) the government's budget constraints are satisfied, (iv) markets clear, and (v), wages and multipliers satisfy the complementary slackness conditions.

Following definition (1), an equilibrium with downward rigid wages is characterized by conditions (3) - (16). The next section formulates the optimal taxation problem with downward rigid wages and characterizes the optimal fiscal policy.

3 Optimal Taxes

In order to characterize the optimal Ramsey income taxes, we start by characterizing the set of attainable allocations. Our approach follows Lucas and Stokey (1983).

Proposition 1 (Set of attainable allocations)**.** An allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$ can be attained as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$ if and only if it satisfies the resource constraints (1) for all (t, s^t), the implementability condition

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[u'(c(s^t))c(s^t)-\theta h'(n(s^t))n(s^t)]\geq \mathcal{W}_0, \tag{17}$$

where

$$\mathcal{W}_0 = u'(c(s^0))\{[(1-\delta+(1-\tau_k(s^{-1}))F_k(k(s^{-1}),n(s^0))]k(s^{-1})+b(s^{-1})\},$$

and the following constraints on the marginal product of labor

$$\begin{aligned} & A(s^t)F_n(k(s^{t-1}),n(s^t))\geq \gamma A(s^{t-1})F_n(k(s^{t-2}),n(s^t)), \quad \forall(t\geq 1,s^t) \\ & \text{and} \quad A(s^0)F_n(k(s^{-1}),n(s^0))\geq \gamma w(s^{-1}) \quad \text{for} \quad t=0. \end{aligned} \tag{18}$$

Proof. See Appendix A. ■

Proposition (1) shows that an attainable allocation is characterized by the resource constraints (1) and the implementability condition (17). These are the same conditions we obtain in an environment with flexible wages. But wages are downward rigid in our environment and a benevolent social planner takes the pecuniary externality into consideration. That is, a benevolent social planner takes into account that the social marginal cost of a higher wage at some history s^t, $w(s^t)$, includes the fact that the wage at some history s^{t+1} that follows from s^t may not be able to fully adjust downwards. This is why the constraints on the marginal product of labor (18) are part of the characterization of attainable allocations.

The Ramsey allocation - the best allocation in the set of attainable allocations - is the allocation that maximizes (2) subject to conditions (1), (17), and (18), given initial conditions $(k(s^{-1}),b(s^{-1}),\tau_k(s^{-1}),w(s^{-1}))$. We assume that initial wealth in utility terms, $\mathcal{W}_0$, is exogenous. The interpretation is that the government cannot confiscate initial wealth, neither directly nor indirectly. In an environment with flexible wages, this assumption ensures that the Ramsey allocation never has inter-temporal distortions, i.e., the optimal ex-ante capital income tax equals zero forever[9]. This assumption also ensures that the Ramsey allocation with flexible wages has the same intra-temporal wedge every period. The proposition that follows shows that these results do not hold when wages are downward rigid.

Proposition 2 (Optimal Ramsey taxes)**.** Let $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$ be a Ramsey allocation with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}), \mathcal{W}_0)$. The state-contingent labor income taxes and ex-ante capital income taxes that implements the Ramsey allocation are, respectively,

$$\tau_n(s^t) = 1 - \frac{1+\mu(1-\sigma)}{1+\mu(1+\psi)} + \beta\gamma \frac{F_{nn}(k(s^{t-1}), n(s^t))}{F_n(k(s^{t-1}), n(s^t))} \frac{\mathbb{E}_t[\eta(s^{t+1})]}{u'(c(s^t))[1+\mu(1+\psi)]} - \frac{F_{nn}(k(s^{t-1}), n(s^t))}{F_n(k(s^{t-1}), n(s^t))} \frac{\eta(s^t)}{u'(c(s^t))[1+\mu(1+\psi)]} \quad \forall (t, s^t) \tag{19}$$

and

$$\tau_k(s^t) = \frac{\beta\gamma \mathbb{E}_t[A(s^{t+1}) F_{nk}(k(s^t), n(s^{t+1})) \mathbb{E}_{t+1}[\eta(s^{t+2})]]}{[1+\mu(1-\sigma)]\mathbb{E}_t[u'(c(s^{t+1}))A(s^{t+1})F_k(k(s^t), n(s^{t+1}))} - \frac{\mathbb{E}_t[A(s^{t+1}) F_{nk}(k(s^t), n(s^{t+1}))\eta(s^{t+1})]}{[1+\mu(1-\sigma)]\mathbb{E}_t[u'(c(s^{t+1}))A(s^{t+1})F_k(k(s^t), n(s^{t+1}))} \quad \forall (t, s^t), \tag{20}$$

where $\mu > 0$ is the Lagrange multiplier on the implementability condition (17), and $\beta^t \pi(s^t)\eta(s^t) \geq 0$ is the present value Lagrange multiplier on the constraint on the marginal product of labor (18) at history s^t.

[9] This result depends on the assumption that preferences are standard preferences in macroeconomics, as they are in our environment. For a more detailed discussion, see Chari et al (2018).

Proof. See Appendix B. ■

Proposition (2) shows how downward rigid wages affect the optimal state-contingent labor income tax and the optimal ex-ante capital income tax. The optimal state-contingent labor income tax (19) is not constant over the business cycle, even under the assumption that preferences are standard preferences in macroeconomics. To see how the optimal labor income tax moves over the business cycle, consider some history s^t such that the Ramsey allocation has $\eta(s^t) = 0$. The optimal labor income tax is

$$\tau_n(s^t) = 1 - \frac{1+\mu(1-\sigma)}{1+\mu(1+\psi)} + \beta\gamma \frac{F_{nn}(k(s^{t-1}), n(s^t))}{F_n(k(s^{t-1}), n(s^t))} \frac{\mathbb{E}_t[\eta(s^{t+1})]}{u'(c(s^t))[1+\mu(1+\psi)]}.$$

The second term in this equation exists because of the nature of the wage rigidity. The benevolent social planner takes into consideration that the choice for the marginal product of labor at history s^t affects the possible choices for the marginal product of labor for all histories s^{t+1} that follow from s^t. Since the downward wage rigidity constraint (3) is not binding at history s^t, optimal policy consists in using the labor income tax to induce a lower equilibrium wage $w(s^t)$. This requires giving households an incentive to increase the quantity of labor they want to work with a lower labor income tax rate[10]. By reducing the equilibrium wage at history s^t, the benevolent social planner is effectively relaxing the constraints on the marginal product of labor (18) for all histories s^{t+1} that follow from s^t. If, on the other hand, history s^t is such the Ramsey allocation has $\eta(s^t) > 0$, this mechanism is inaccessible because the wage is at the lower bound and cannot adjust downwards. In this case, the labor market outcome is determined by the firms' demand (10). Despite choosing the quantity of labor that solves the profit-maximization problem, households are willing to sacrifice some resources to be able to work a little more. That

[10] Recall that the production function $F(\cdot)$ is strictly increasing and strictly concave in both its arguments.

is, the constraint (5) is binding. Optimal policy consists in increasing the labor income tax rate in such a way that the constraint (5) is not binding at the solution to the utility-maximization problem at history s^t. This is because an increase in the labor income tax rate when the downward wage rigidity constraint (3) is binding does not change the labor market outcome and, as such, results in an increase in revenue without raising additional distortions.

The optimal ex-ante capital income tax (20) is not equal to zero over the business cycle. To see how the optimal capital income tax moves over the business cycle, first observe that it only depends on expectations about the future. Now, consider some history s^t where $\eta(s^{t+1}) > 0$ for some history s^{t+1} that follows from s^t. This means that the constraint on the marginal product of labor (18) is binding at some history s^{t+1} that follows from s^t. Holding everything else constant, optimal policy consists in decreasing the capital income tax rate that is to be collected at s^{t+1}. The interpretation is that a lower ex-ante capital income tax increases investment at s^t, leading to a higher stock of capital at all histories s^{t+1} that follow from s^t. Since the marginal product of labor depends positively on the stock of capital, this policy results in a higher marginal product of capital at all histories s^{t+1}, relaxing the downward wage rigidity constraints. This idea is captured in the second line of (20). Next, consider that the Ramsey allocation has $\eta(s^{t+2}) > 0$ for some history s^{t+2} that follows from s^t, meaning that the constraint on the marginal production of labor (18) is binding at some history s^{t+2} that follows from s^t. In this case, optimal policy consists in increasing the capital income tax rate collected at all histories s^{t+1} that follow from s^t. This has to do with the fact that the lower bound on the wage at all histories s^{t+2} depends on the realized wage $w(s^{t+1})$. A higher capital income tax rate $\tau_k(s^t)$ reduces investment and leads to a lower stock of capital at all histories s^{t+1} that follow from s^t. Everything else constant, a lower stock of capital results in a lower equilibrium wage $w(s^{t+1})$, which effectively relaxes the downward wage rigidity constraints (3) for all histories s^{t+2} that follow from s^t. This idea is captured by the first line of (20). Finally, observe that this analysis depends on the assumption that the production function is a standard neoclassical production function with $F_{nk}(\cdot) > 0$. If labor and capital were not related in production, i.e., $F_{nk}(\cdot) = 0$, the optimal ex-ante capital income tax would be zero for all (t, s^t).

In the next section, we perform a numerical exercise to see the mechanisms described in (19) and (20) and discussed above. This exercise is relevant because we related these mechanisms to the cyclical behavior of the optimal Ramsey taxes.

4 Numerical Exercise

We start this section by introducing the recursive formulation of Ramsey problem. This is a relatively straightforward exercise, but we want to emphasize some particular aspects of the formulation as well as the role of some of our assumptions.

4.1 Recursive formulation of the Ramsey problem

Denote by $\mathcal{W}(s^t)$ the value of wealth in utility terms at history s^t. This means that

$$\mathcal{W}(s^t) = u'(c(s^t))\{[1 - \delta + (1 - \tau_k(s^{t-1}))F_k(k(s^{t-1}), n(s^t))]k(s^{t-1}) + b(s_t|s^{t-1})\}.$$

We can use this variable to write the intra-temporal implementability constraints in a recursive fashion:

$$u'(c(s^t))c(s^t) - \theta h'(n(s^t))n(s^t) + \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t)\mathcal{W}(s_{t+1}|s^t) =$$
$$\mathcal{W}(s^t) + \lambda(s^t)n(s^t) + u'(c(s^t))T(s^t) \quad \forall (t, s^t).$$

This allows us to treat the value of wealth in utility terms as a state variable. As explained in Chari, Nicolini, and Teles (2020), promises of wealth in utility terms constrain the policy choices of the benevolent social planner. Furthermore, our assumption that initial wealth in utility terms, $\mathcal{W}_0$, is exogenous and the consideration that the government has access to lump-sum transfers at all (t, s^t) make all periods look alike. This is a simplifying assumption that eliminates the difference between period zero and all future periods, which is standard in the literature of optimal Ramsey taxation. Therefore, the recursive formulation of our Ramsey problem consists of only one Bellman equation.

The constraint on the marginal product of labor (18) suggests that we must keep track of previous shock, stock of capital, and labor. However, we find it easier to keep track of the realization of the previous wage, since it includes all this information. We are effectively adding an additional sequence of choice variables to the Ramsey problem, $\{\{w(s^t)\}_{s^t}\}_{t \geq 0}$, as well as an additional sequence of constraints,

$$w(s^t) = A(s^t)F_n(k(s^t), n(s^t)) \quad \forall (t, s^t).$$

Observe that the Ramsey allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t \geq 0}$ also solves the Ramsey problem augmented by the introduction of these additional constraints.

The benevolent social planner enters every period with a predetermined stock of capital, k, the previous realization of the wage, w_{-1}, and a vector of wealth in utility terms, $\{\mathcal{W}(s)\}_{s \in S}$. Before making any decisions, the benevolent social planner observes the realization of the random variable $s \in S$, which determines the value of aggregate productivity, $A(s)$, government expenditures, $g(s)$, and the wealth in utility terms $\mathcal{W}(s)$ that must be delivered this period. This means that the state vector is $(s, k, w_{-1}, \mathcal{W}(s))$. After observing the realization of the exogenous state s, the benevolent social planner chooses consumption c, labor n, wage w, the stock of capital for next period k', and the vector of promised wealth in utility terms $\{\mathcal{W}'(s')\}_{s' \in S}$. Let $V(s, k, w_{-1}, \mathcal{W}(s))$ denote the value associated with the Ramsey plan $(c, n, k', w, \{\mathcal{W}'(s')_{s' \in S})$ at state $(s, k, w_{-1}, \mathcal{W}(s))$. This value satisfies the Bellman equation[11,12]

$$
\begin{aligned}
V(s, k, w_{-1}, \mathcal{W}) = \max_{(c,n,k',w,\{\mathcal{W}'(s')_{s' \in S})} & \; u(c) - \theta h(n) + \beta \sum_{s' \in S} \pi(s'|s) V(s', k', w, \mathcal{W}'(s')) \\
\text{s.t.} \quad & c + g(s) + k' - (1-\delta)k = A(s)F(k,n) \\
& u'(c)c - \theta h'(n)n + \beta \sum_{s' \in S} \pi(s'|s) \mathcal{W}'(s') = \mathcal{W} \\
& w = A(s)F_n(k,n) \\
& w \geq \gamma w_{-1}.
\end{aligned}
$$

[11] The assumption that $\mathcal{W}_0$ is large enough so that the the implementability condition (17) is binding at the solution to the Ramsey problem implies that the Ramsey allocation has $\lambda(s^t) = T(s^t) = 0$ for all (t, s^t). We simplify the Bellman equation by imposing this result and removing the Lagrange multiplier and lump-sum transfers from the implementability constraint.

[12] The numerical exercise, whose results are displayed below in subsection 4.2, are an approximation and they are not obtained from solving the Bellman equation discussed. Instead, we exogenously fix the value of the multiplier of the implementability constraint at a constant level and solve the problem

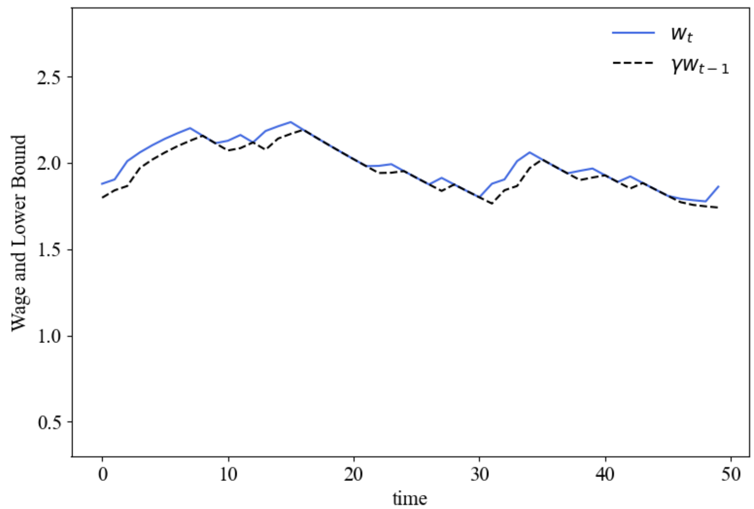
$$
\begin{aligned}
\tilde{V}(s, k, w_{-1}) = \max_{(c,n,k',w)} & \; u(c) - \theta h(n) + \mu \left[u'(c)c - \theta h'(n)n \right] + \beta \sum_{s' \in S} \pi(s'|s) \tilde{V}(s', k', w) \\
\text{s.t.} \quad & c + g(s) + k' - (1-\delta)k = A(s)F(k,n) \\
& w = A(s)F_n(k,n) \\
& w \geq \gamma w_{-1}.
\end{aligned}
$$

We are currently working on the implementation of the Bellman equation, but we do not expect the qualitative results differ from those introduced below.

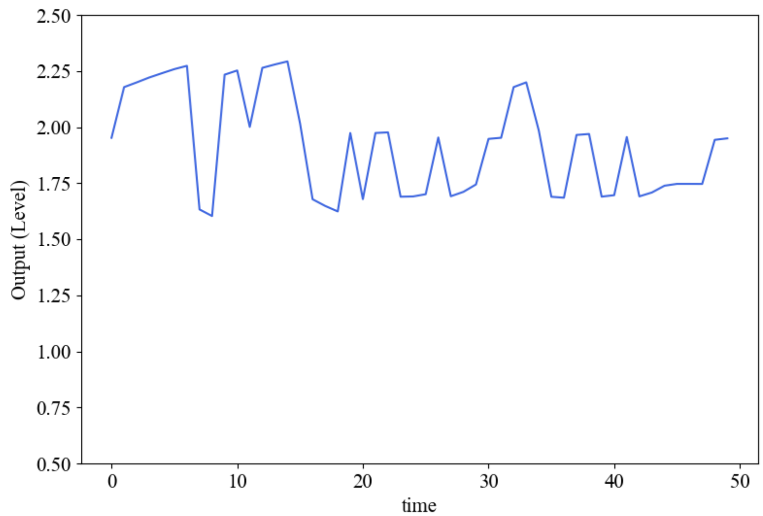
4.2 Cyclical properties of the optimal Ramsey taxes

Figure 1 shows the simulated time series for wages in panel *(a)* and output in panel *(b)*. We include the simulated time series for the endogenous lower bound on wages to clearly show when the downward wage rigidity constraint (3) is binding. The fluctuations in output are due to changes in the realization of the productivity shock. However, these fluctuations are amplified by the presence of downward rigid wages. In particular, periods in which the wage cannot fully adjust downwards are periods in which the decrease in output exceeds the change we would observe if wages were flexible. The reason is that if the wage cannot fully adjust downwards in response to a low realization of the productivity shock, the quantity of labor worked, determined by firms' demand, is lower than if wages were flexible.

In the previous section, we discussed the two opposite forces that govern the dynamics of the optimal state-contingent labor income tax (19). The nature of the wage rigidity - the fact that lower bound on the wage is endogenous - makes it optimal to have a low labor income tax rate to induce a low equilibrium wage at (t, s^t) and, consequently, a low floor on the wage for the state s^{t+1} that follows. However, in periods where the wage cannot fully adjust downwards, it is optimal to have a labor income tax high enough so that the rationing constraint (5) is not binding at the solution to the utility-maximization problem. In doing so, the government explores the opportunity to collect revenue without introducing additional distortions.



(a) Time series for wages



(b) Time series for output

Figure 1: Simulated path for wages and output

Notes: We solve the Bellman equation presented in footnote 11 and simulate a time series for the aggregate productivity shock with 100,000 observations, where one period in our simulation is intended to represent one quarter. The figure presents the 50 observations within the interval 90,000 - 90,049. The time series for wages in panel *(a)* is obtained from the policy function for the wage. Panel *(a)* also includes the time series for the lower bound on the wage in every period. We obtain the lower bounds as seen in the downward wage rigidity constraint (3). The time series in panel *(b)* is obtained from the policy functions for capital and labor.

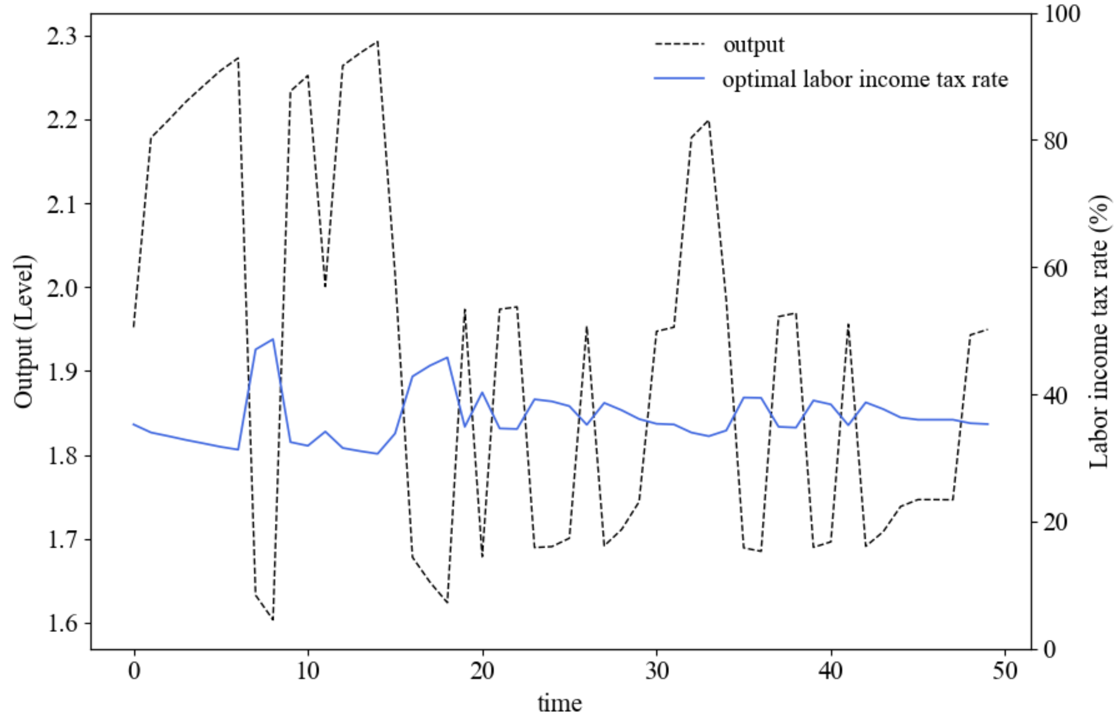


Figure 2: Counter-cyclical behavior of optimal labor income tax

Notes: We solve the Bellman equation presented in footnote 11 and simulate a time series for the aggregate productivity shock with 100,000 observations, where one period in our simulation is intended to represent one quarter. The figure presents the 50 observations within the interval 90,000 - 90,049. The left vertical axis displays output levels and the right vertical axis displays labor income tax rates. The time series for output is derived from the policy functions for capital and labor. To obtain the time series for the labor income tax rate, we use equation (8) with $\lambda(s^t) = 0$.

Figure 2 plots the the simulated time series for output (left vertical axis) and the simulated time series for the optimal labor income tax rate (right vertical axis) and it shows that the optimal labor income tax rate is counter-cyclical. Indeed, Figure 2 shows that the optimal labor income tax increases whenever output decreases. The interpretation of this cyclical behavior is as discussed above. Following a low productivity shock that drives the wage to the lower bound - and where the wage would further decrease if wages were flexible; that is, where (3) is binding - output decreases and this effect is augmented by the fact that the wage cannot fully adjust downwards. Moreover, the additional constraint on the consumption set (5) is binding without government policy. This means that households are willing to pay a positive price to relax this constraint and work more, but the labor market outcome is determined by the firms' demand. Therefore, a higher labor income tax does not change the labor market outcome. Instead, it increases tax revenue without introducing any additional distortions. This makes it optimal to increase the labor income tax.

In our environment, the optimal labor income tax ensures that the constraint (5) is never binding at the solution to the utility-maximization problem, not even in states where the the wage is at its lower bound. That is, the optimal labor income tax ensures that $\lambda(s^t) = 0$ for all (t, s^t). This does not mean, however, that optimal policy eliminates the effect of downward rigid wages. If that were true, the allocation would be the same as under flexible wages and this is not the case. Indeed, when the downward wage rigidity constraint (3) is binding, the labor market outcome is characterized by a quantity of labor below the flexible-wage counterpart. The reason is that the government does not have access to a fiscal instrument that can reduce the effective cost of labor to firms and increase the quantity of labor demanded at the market wage rate. An example of such a fiscal instrument is a state-contingent payroll tax and we can show that a combination of lower payroll taxes and higher labor income taxes would eliminate the effects of downward rigid wages[13]. Moreover, the implication of optimal policy in our environment is that the intra-temporal marginal conditions (8) become

$$\frac{\theta h'(n(s^t))}{u'(c(s^t))} = (1 - \tau_n(s^t))w(s^t) \quad \forall (t, s^t).$$

[13]With payroll taxes, the wage $w(s^t)$ is no longer the relevant cost of labor to firms. Instead, the effective cost of labor is $(1 + \tau_p(s^t))w(s^t)$, where $\tau_p(s^t)$ is the payroll tax rate at history s^t. Payroll taxes allow the government to control the first-order condition (10), which makes it possible to induce firms to hire any arbitrary quantity of labor, even when the downward wage rigidity constraint (3) is binding. See Adão, Correia, and Teles (2008, 2009) and Farhi, Gopinath, and Itskhoki (2014) for a detailed discussion of how fiscal policy can be used to implement flexible-price allocations when prices are sticky.

Under the optimal labor income tax policy, the intra-temporal wedge is akin to the definition found in the literature of business cycle accounting - the ratio of the marginal rate of substitution between labor and consumption to the marginal product of labor. Our results, although normative, are not unrealistic. Shimer (2009), for example, computes the labor wedge for the United States and shows that it is counter-cyclical.

In section 3, we also discussed the two opposite forces that govern the dynamics of the optimal ex-ante capital income tax (20). The fact that a downward rigid wage consists of a pecuniary externality makes it optimal to choose a high capital income tax at s^t. This is because it reduces the stock of capital in the period in which the capital income tax is collected. Consequently, the wage is low in this period, leading to a low floor on the wage at all s^{t+2} that follow from s^t. However, the expectation that the wage will hit the lower bound at some s^{t+1} that follows from s^t makes it optimal to choose a low capital income tax at s^t, because the capital stock for all s^{t+1} increases. This, in turn, leads to a higher wage in period in which the ex-ante capital income tax is collected, reducing the negative effects of the downward wage rigidity constraint (3) at all s^{t+1}.

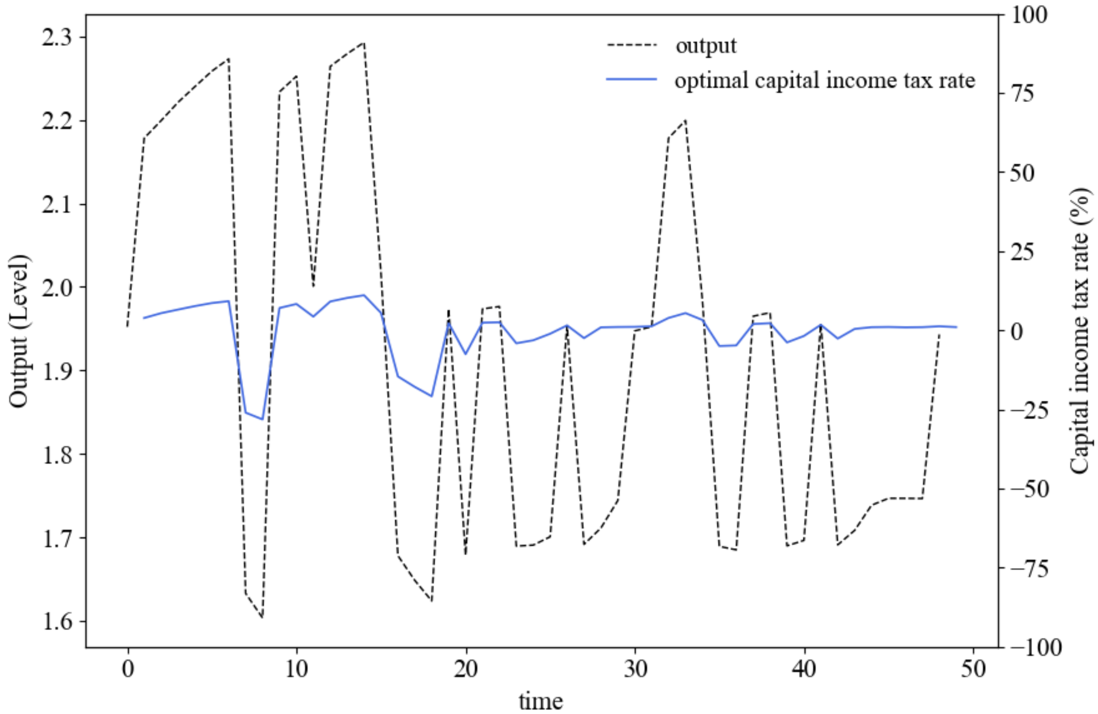


Figure 3: Pro-cyclical behavior of optimal capital income tax

We solve the Bellman equation presented in footnote 11 and simulate a time series for the aggregate productivity shock with 100,000 observations, where one period in our simulation is intended to represent one quarter. The figure presents the 50 observations within the interval 90,000 - 90,049. The left vertical axis displays output levels and the right vertical axis displays capital income tax rates. The time series for output is derived from the policy functions for capital and labor. We compute the time series for the ex-ante capital income tax rate using the policy function for consumption and the inter-temporal marginal conditions (6).

Figure 3 plots the simulated time series for both output (right vertical axis) and the optimal ex-ante capital income tax (left vertical axis). We can see that the optimal ex-ante capital income tax exhibits a pro-cyclical behavior. Indeed, the optimal ex-ante capital income tax rate decreases when output decreases. The interpretation of this result is as discussed above. In response to a low productivity shock that makes the downward wage rigidity constraint (3) binding, output decreases. This low realization of the productivity shock was given a positive probability in the previous period. A lower ex-ante capital income tax leads to a higher marginal product of labor when the productivity shock is low, thus making the effects of downward rigid wages less severe. Therefore, it is optimal to decrease the ex-ante capital income tax rate when output decreases.

5 Conclusion

In this paper, we study the implications on optimal income taxation of downward rigid wages. We focus on a standard neoclassical economy and impose a friction in the labor market that makes it impossible for the real wage to fall below some endogenously determined lower bound. The endogeneity of this lower bound introduces a pecuniary externality since private agents fail to recognize the effect that their current decisions have on the lower bound in the future. In this environment, the classical results in the literature of optimal taxation no longer hold. In particular, the optimal state-contingent labor income tax rate is not constant over time and the optimal ex-ante capital income tax is not equal to zero in every period, even if preferences are standard preferences in macroeconomics. The reason is that these taxes can be used to alleviate the effects of downward rigid wages, although they are not sufficient to completely eliminate these effects and implement the flexible wage allocation.

The goal of optimal policy in this environment is to decrease the amplitude of the fluctuations of the equilibrium wage in states of the world where the wage is above the lower bound. By decreasing the equilibrium wage in such a state, we reduce the exposure to the endogenously determined lower bound on wages in the future. To accomplish this goal, we perform a numerical exercise and show that the optimal labor income tax acquires counter-cyclical properties and that the optimal capital income tax acquires pro-cyclical properties.

Furthermore, a predominant feature of models with wage rigidities is the existence of involuntary unemployment. Whenever the wage in a particular state is at the lower bound, households are willing to pay a positive price in order to increase the quantity of labor worked above the quantity firms want to hire. In order words, the multiplier on the rationing constraint is positive. However, we show that optimal policy always eliminates involuntary unemployment. In such a state of the world, a higher labor income tax decreases the quantity of labor households want to supply at each pre-tax wage and, by choosing the correct labor income tax, the quantity of labor households want to supply equals the quantity of labor firms demand at a pre-tax wage equal to the lower bound. Nevertheless, this quantity of labor is below the flexible wage counterpart.

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Appendix

A Proof of Proposition 1

Our proof of Proposition (1) starts with the two Lemmas that follow.

Lemma 1. An allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$, Lagrange multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ can be attained as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$ if and only if they satisfy the resource constraints (1) for all (t, s^t), the constraints on the marginal product of labor (18), the implementability condition

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[u'(c(s^t))c(s^t) - \theta h'(n(s^t))n(s^t)] = \mathcal{W}_0 + \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[\lambda(s^t)n(s^t) + u'(c(s^t))T(s^t)], \tag{21}$$

and the complementary slackness conditions

$$\begin{aligned}
&\lambda(s^t)[A(s^t)F_n(k(s^{t-1}), n(s^t)) - \gamma A(s^{t-1})F_n(k(s^{t-2}), n(s^{t-1}))] = 0, \quad \forall(t \geq 1, s^t)\\
&\lambda(s^0)[A(s^0)F_n(k(s^{-1}), n(s^0)) - \gamma w(s^{-1})] = 0 \quad \text{for} \quad t = 0,\\
&\lambda(s^t) \geq 0, \quad \forall(t, s^t).
\end{aligned} \tag{22}$$

The implementability condition (21) suggests that lump-sum transfers $T(s^t)$ and Lagrange multipliers $\lambda(s^t)$ play a similar role in the characterization of attainable allocations. To see this, consider an equilibrium with downward rigid wages where $\lambda(s^t) > 0$ for some history s^t. This implies that the labor income tax rate at history s^t is such that

$$\frac{\theta h'(n^t)}{u'(c(s^t))} < (1-\tau_n(s^t))w(s^t).$$

An alternative implementation of the same allocation consists of a higher labor income tax rate $\tau_n(s^t)$ such that the marginal rate of substitution between labor and consumption equals the after-tax wage and a higher lump-sum transfer. This alternative allocation has a Lagrange multiplier equal to zero. Therefore, households are effectively receiving a lump-sum transfer when $\lambda(s^t) > 0$. This intuition leads to the following Lemma[14].

Lemma 2. Let the allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$, multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ be attainable as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$. There exists a sequence of lump-sum transfers $\{\{\hat{T}(s^t)\}_{s^t}\}_{t\geq 0}$ with $\hat{T}(s^t) \geq T(s^t)$ for all (t, s^t) such that the allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$ and lump-sum transfers $\{\{\hat{T}(s^t)\}_{s^t}\}_{t\geq 0}$ are also attainable as part of an equilibrium with downward rigid wages with the initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$ where the Lagrange multipliers are $\hat{\lambda}(s^t) = 0$ for all (t, s^t).

Proof of Lemma 1. In one direction, take an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$. Conditions (3) - (16) are satisfied and we must show that (1), (18), (21), and (22). Use the input market clearing conditions (14) and (15) to substitute away the firm allocation from the equilibrium conditions. Doing this in the output market clearing conditions (13) yields (1). Doing this in the firms' first-order condition (10) yields the wage $w(s^t)$ in terms of $(k(s^{t-1}), n(s^t))$. Next, substitute away the resulting equation in the downward wage rigidity constraints (3) and

[14]Our environment allows for lump-sum transfers because we want to make allocations attainable with $\lambda(s^t) = 0$ for all (t, s^t). Without lump-sum transfers, this would only be necessarily true for the allocation that solves the optimal taxation problem

the complementary slackness conditions (16). This yields (18) and (22), respectively. The households' consolidated budget constraint is

$$\sum_{t=0}^{\infty}\sum_{s^t}(\prod_{j=0}^{t}q(s_j|s^{j-1}))[c(s^t)-(1-\tau_n(s^t))w(s^t)n(s^t)] =$$
$$(1-\delta+(1-\tau_k(s^{-1}))r(s^0))k(s^{-1})+b(s^{-1})+\sum_{t=0}^{\infty}\sum_{s^t}(\prod_{j=0}^{t}q(s_j|s^{j-1}))T(s^t),$$

where $q(s_0|s^{-1}) = 1$. Use the Euler equation (7) to substitute away the prices of state-contingent assets, the intra-temporal marginal conditions (8) to substitute away after-tax wages, and the firms' first-order conditions (11) to substitute away the capital rent at $t = 0$. This yields (21).

In the other direction, take an allocation allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$, multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ that satisfy conditions (1), (18), (21), and (22). We need to find a firm allocation, state-contingent assets, prices, and policies, such that conditions (3) - (16) are satisfied. The input market clearing conditions (14) and (15) pin down the firm allocation $\{\{k^d(s^t), n(s^t)\}_{s^t}\}_{t\geq 0}$. The firms' first-order conditions (10) and (11) pin down pre-tax wages $\{\{w(s^t)\}_{s^t}\}_{t\geq 0}$ and capital rents $\{\{r(s^t)\}_{s^t}\}_{t\geq 0}$, respectively. The Euler equations (6) and (7) pin down ex-ante capital income tax rates $\{\{\tau_k(s^t)\}_{s^t}\}_{t\geq 0}$ and state-contingent asset prices $\{\{q(s_{t+1}|s^t)\}_{s^t}\}_{t\geq 0}$, respectively. The intra-temporal marginal conditions (8) pin down labor income tax rates $\{\{\tau_n(s^t)\}_{s^t}\}_{t\geq 0}$. The households' budget constraints (4) pin down state-contingent assets $\{\{b(s_{t+1}|s^t)\}_{s^t}\}_{t\geq 0}$. The complementary slackness conditions that characterize the solution to the utility-maximization problem (9) are satisfied since (15) holds. The output market clearing conditions (13) are satisfied since (1) holds, and since (14) and (15) hold. The downward wage rigidity constraints (3) complementary slackness conditions

(16) are satisfied since (18) and (22) hold. Finally, the government's budget constraints (12) are satisfied by Walras' law. ■

Proof of Lemma 2. Take an allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$, multipliers $\{\{\lambda(s^t)\}_{s^t}\}_{t\geq 0}$, and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ that can be attained as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$. It follows from Lemma (1) that conditions (1), (18), (21), and (22) are satisfied. Now, define the new sequence of lump-sum transfers, $\{\{\hat{T}(s^t)\}_{s^t}\}_{t\geq 0}$ as follows:

$$\hat{T}(s^0) = T(s^0) + \frac{1}{u'(c(s^0))}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)\lambda(s^t)n(s^t)$$
$$\hat{T}(s^t) = T(s^t), \quad \forall (t \geq 1, s^t).$$

It suffices to show that the allocation, the new sequence of lump-sum transfers, and the sequence of multipliers $\{\{\hat{\lambda}(s^t)\}_{s^t}\}_{t\geq 0}$, where $\hat{\lambda}(s^t) = 0$ for all (t, s^t) satisfy conditions (1), (18), (21), and (22). The resource constraints (1) and the constraints on the marginal product of labor (18) are satisfied, since the allocation is attainable as an equilibrium with downward rigid wages. Conditions (22) are satisfied with $\hat{\lambda}(s^t) = 0$ for all (t, s^t). To see that the implementability condition (21) is satisfied, observe that

$$\begin{aligned}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[\hat{\lambda}(s^t)n(s^t) + u'(c(s^t))\hat{T}(s^t)] &= \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)u'(c(s^t))\hat{T}(s^t)\\ &= u'(c(s^0))[T(s^0) + \frac{1}{u'(c(s^0))}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)\lambda(s^t)n(s^t)] +\\ &\quad \sum_{t=1}^{\infty}\sum_{s^t}\beta^t\pi(s^t)u'(c(s^t))T(s^t)\\ &= \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[\lambda(s^t)n(s^t) + u'(c(s^t))T(s^t)].\end{aligned}$$

This completes the proof. ■

Lemma (1) and Lemma (2) yield the following corollary.

Corollary. An allocation $\{\{c(s^t), n(s^t), k(s^t)\}_{s^t}\}_{t\geq 0}$ and lump-sum transfers $\{\{T(s^t)\}_{s^t}\}_{t\geq 0}$ can be attained as part of an equilibrium with downward rigid wages with initial conditions $(k(s^{-1}), b(s^{-1}), \tau_k(s^{-1}), w(s^{-1}))$ if and only if they satisfy the resource constraints (1) for all (t, s^t), constraints on the marginal product of labor (18) and the implementability condition

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[u'(c(s^t))c(s^t) - \theta h'(n(s^t))n(s^t)] = \mathcal{W}_0 + \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)u'(c(s^t))T(s^t). \quad (23)$$

To complete the proof of Proposition (1), we need only to impose the non-negativity of lump-sum transfers.

B Proof of Proposition 2

Let $\mu > 0$ be the Lagrange multiplier on (17) and $\beta^t\pi(s^t)\eta(s^t) \geq 0$ be the present value Lagrange multiplier on (18) at history s^t. The Ramsey allocation must satisfy the necessary first-order conditions

$$\begin{aligned}\frac{\theta h'(n(s^t))}{u'(c(s^t))} =& \frac{1+\mu(1-\sigma)}{1+\mu(1+\psi)}A(s^t)F_n(k(s^{t-1}), n(s^t)) + \frac{A(s^t)F_{nn}(k(s^{t-1}), n(s^t))}{u'(c(s^t))[1+\mu(1+\psi)]}\eta(s^t) - \\ & \beta\gamma\frac{A(s^t)F_{nn}(k(s^{t-1}), n(s^t))}{u'(c(s^t))[1+\mu(1+\psi)]}\sum_{s_{t+1}|s^t}\pi(s_{t+1}|s^t)\eta(s^{t+1})\end{aligned}$$

and

$$
\begin{aligned}
u'(c(s^t)) =& \beta \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) u'(c(s^{t+1}))[1-\delta + A(s^{t+1}) F_k(k(s^t), n(s^{t+1}))] + \\
& \beta \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) \frac{A(s^{t+1}) F_{nk}(k(s^t), n(s^{t+1}))}{1+\mu(1-\sigma)} \eta(s^{t+1}) - \\
& \beta^2 \gamma \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) \frac{A(s^{t+1}) F_{nk}(k(s^t), n(s^{t+1}))}{1+\mu(1-\sigma)} \sum_{s_{t+2}|s^{t+1}} \pi(s_{t+2}|s^{t+1}) \eta(s^{t+2})
\end{aligned}
$$

for all (t, s^t). An equilibrium allocation with downward rigid wages must satisfy, for all (t, s^t),

$$
\frac{\theta h'(n(s^t))}{u'(c^t))} = (1-\tau_n(s^t)) A(s^t) F_n(k(s^{t-1}), n(s^t)) - \frac{\lambda(s^t)}{u'(c(s^t))}
$$

and

$$
u'(c(s^t)) = \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) u'(c(s^{t+1}))[1-\delta + (1-\tau_k(s^t)) A(s^{t+1}) F_k(k(s^t), n(s^{t+1}))].
$$

Lemma (2) shows that the Ramsey allocation can be implemented with $\lambda(s^t) = 0$ for all s^t. Therefore, the state-contingent labor income tax at history s^t that implements the Ramsey allocation is implicitly defined by

$$
\begin{aligned}
1 - \tau_n(s^t) =& \frac{1+\mu(1-\sigma)}{1+\mu(1+\psi)} + \frac{F_{nn}(k(s^{t-1}), n(s^t))}{F_n(k(s^{t-1}), n(s^t))} \frac{\eta(s^t)}{u'(c(s^t))[1+\mu(1+\psi)]} - \\
& \frac{\beta\gamma}{u'(c(s^t))[1+\mu(1+\psi)]} \frac{F_{nn}(k(s^{t-1}), n(s^t))}{F_n(k(s^{t-1}), n(s^t))} \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) \eta(s^{t+1}).
\end{aligned}
$$

Solving for $\tau_n(s^t)$ yields (19). And the ex-ante capital income tax at history s^t that implements the Ramsey allocation is implicitly defined by

$$\tau_k(s^t))\beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)u'(c(s^{t+1}))A(s^{t+1})F_k(k(s^t),n(s^{t+1})) =$$

$$\beta \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t)\frac{A(s^{t+1})F_{nk}(k(s^t),n(s^{t+1}))}{1+\mu(1-\sigma)}\eta(s^{t+1}) -$$

$$\beta^2\gamma \sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t)\frac{A(s^{t+1})F_{nk}(k(s^t),n(s^{t+1}))}{1+\mu(1-\sigma)} \sum_{s_{t+2}|s^{t+1}} \pi(s_{t+2}|s^{t+1})\eta(s^{t+2}).$$

Solving for $\tau_k(s^t)$ yields (20).

This completes the proof of Proposition (2).

C Functional forms, parameters, and exogenous shocks

In this section, we present the functional forms used in our numerical exercise, as well as the parameter values and the structure of the shocks.

C.1 Functional forms

The functional forms chosen for the numerical exercise are as follows. The utility function is

$$u(c) - \theta h(n) = \frac{c^{1-\sigma}}{1-\sigma} - \theta\frac{n^{1+\psi}}{1+\psi},$$

where σ represents the inter-temporal elasticity of substitution for consumption, and ψ represents the inverse of the Frisch elasticity of labor supply. With preferences represented by such an utility function, the traditional results in the optimal taxation literature hold: the optimal state-contingent labor income tax is constant over time and the optimal ex-ante capital income tax converges to zero if the government can confiscate initial wealth indirectly or is exactly equal to zero starting from period 0 otherwise.

The production function is a classic neoclassical production function of the Cobb-Douglas type,

$$F(k,n) = k^{\alpha} n^{1-\alpha},$$

where α denotes the capital income share.

C.2 Parameter values and exogenous shocks

The parameter values used in the numerical exercise as well as the values of the productivity shocks and the corresponding transition matrix were all chosen externally. The table below summarizes the parameter values and discusses their meaning.

Parameter	Description	Value
β	subjective discount factor	0.98
σ	intertemporal elasticity of substitution for c	2
ψ	inverse of Frisch elasticity of labor supply	0.25
θ	disutility of labor	1
μ	multiplier of implementability constraint	0.2
α	capital income share	1/3
δ	depreciation rate	0.1
γ	degree of downward wage rigidity	0.98

Regarding the exogenous shocks, we assume, for simplicity, that government spending is constant over time. Allowing for a stochastic process for government spending could also have interesting implications in a model with downward rigid wages associated with pure income effects, which would affect labor supply. But our numerical exercise focuses on productivity shocks only. We assume that government spending is constant and equal to 0.2.

Finally, we assume that productivity A follows a finite Markov process, where the state space is the set $[1.4, 1.5, 1.6]$ and the transition matrix Π is

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.25 & 0.5 & 0.25 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}.$$